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Soil Moisture as a Factor in Streamflow Forecasting for Logan River, Utah

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SOIL MOISTURE AS A FACTOR IN STREAMFLOW

FORECASTING FOR LOGAN RIVER, UTAH

by

Yu Kam Fok

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Yu Kam Fok

TABLE OF CONTENTS

LIST OF TABLES

LIST OF TABLES, continued

LIST OF FIGURES

LIST OF FIGURES, continued

INTRODUCTION

Purpose

Forecasting the annual water supply in an arid area by using the water content of snow on watersheds on some particular date, such as April 1, has become a very useful practice. Although these forecasts have given results of great practical value, they have sometimes been considerably in error. Seeking to minimize error, forecasters have incorporated various additional data such as temperature and antecedent rain to improve the relation between snow measurement and measured runoff.

Numerous methods have been suggested in the search for a reliable streamflow forecasting equation and various data have been used. Nearly all of the methods made some improvements, but in the attempt to minimize the number of variables, perhaps full use has not been made of all the available data.

A successful streamflow forecasting method for Logan River, Cache County, Utah was suggested by Professor Cleve H. Milligan (11) and Dr. Rex L. Hurst*. They utilized Fourier Series and Multiple Linear Regression as a mathematical model. In their study, four primary factors were used which are antecedent streamflow, temperature, precipitation, and snow survey data. This method has also been used in the forecasting for the Blacksmith Fork River, south of the Logan River, by Fok (5) with a high degree of accuracy. In his study, temperature and precipitation data were both measured outside the watershed and showed a lower degree of significance in the complete forecasting equation. If these data had

* Dr. Rex L. Hurst, Head of the Applied Statistics Department, Utah State University, Logan, Utah.

been measured in the watershed they might have yielded greater significance in the forecasting equation. Perhaps a better factor than temperature and precipitation would be soil moisture data obtained on the watershed.

Objective

The major objective of this thesis is to develop a method for use of soil moisture data in an equation for streamflow forecasting for the Logan River in northern Utah. Several investigators have recognized the need for soil moisture data and for a method of including it in the forecasting equations (see literature review).

REVIEW OF LITERATURE

Much has been written concerning the many factors (4) which influence the accuracy of the streamflow forecasting. Most of these contributions to the literature have had to do with precipitation and temperature. Little has been done to determine the amount of water from a given snow-cover which is required to bring the moisture in the earth-mantle under the snow-cover up to field capacity and to hold it there during the melting season. Goodell (6) stated that from nearly one-quarter to over one-third of the potential water yield from the average winter snow accumulation must have been absorbed and retained by the soil in restoring it to field-moisture capacity. Generally, the moisture condition of the soil prior to the snow melt period has an appreciable effect on the quantitative relation between snow accumulation and the yield of water to streamflow. Usually a snow pack on dry soil will not contribute to streamflow as much as if the soil is wet. As indicated by Clyde and Work (3), every watershed is a law unto itself and abnormal fall and spring precipitation, temperature during melting, and soil moisture condition on the watershed affect the basic snow cover runoff relationships. Also Clyde (2) states: "Soil moisture studies are needed as an aid in forecasting runoff from snow cover." Croft (4) said, in part: "A reliable index of available water supply may be obtained by measuring water stored in the soil mantle in addition to that stored as snow about April 1." Also he said, "Winter melting of snow may result in transfer of substantial amounts of water from snowpack to the soil mantle where it cannot be evaluated unless the moisture content of the soil mantle is measured."

Even though most of these investigators mention the importance of soil moisture data and suggest methods for measuring it, they do not show exactly how these data can be included in the forecasting relationships to improve the accuracy of streamflow forecasts.

PROCEDURES

The method described in this thesis utilizes April 1 snow survey data, monthly mean soil moisture, and antecedent monthly streamflow in the forecasting procedure. All of these data are tabulated in tables 1 and 2. Fourier coefficients are utilized to represent antecedent streamflow and soil moisture data. A mathematic model, which is hereafter described, is utilized to give and accurate, unbiased prediction of expected streamflow.

Source of data

Streamflow data: Streamflow data for Logan River were obtained from publication by the U. S. Geological Survey, U. S. Department of Interior in Geological Survey Water-Supply Paper, Part 10, The Great Basin.

Snow survey data: Snow data were taken from U. S. Department of Agriculture, Soil Conservation Service, in "Federal-State Cooperative Snow Surveys and Water Supply Forecasts for Utah." Data from only two snow courses on the Logan River watershed were utilized in this study. These snow courses were Franklin Basin, elevation 8200 feet and Garden City Summit, elevation 7900 feet. These two stations were selected because of high correlation with Logan River runoff and because of consistency of the records.

Soil moisture data: Soil moisture data were obtained by field measurements throughout the year by the electrical resistance method (8) at six soil moisture measurement stations in the Logan River

Table 1. Soil moisture data at Klondike Narrows Station

Year	Months	total d"	1st. W%	foot d"	2nd. W%	foot d"	3rd. W%	foot d"	4th. W%	foot d"	5th. W%	foot d"	6th. W%	foot d"
1957	Oct.	4.46	3.33	0.480	3.53	0.695	2.53	0.498	4.53	0.881	4.93	0.881	5.17	1.024
	Nov.	4.47	5.60	0.806	3.53	0.695	2.38	0.468	4.01	0.779	4.31	0.771	4.79	0.948
	Dec.	4.63	6.73	0.969	5.25	1.033	2.40	0.472	3.63	0.706	3.87	0.692	4.13	0.818
	Jan.	5.01	7.66	1.104	5.70	1.122	2.40	0.472	4.14	0.804	3.80	0.679	4.19	0.829
	Feb.	5.23	8.77	1.263	6.60	1.299	2.53	0.498	3.63	0.706	3.60	0.644	4.13	0.818
	Apr.	9.55	13.63	1.963	13.67	2.690	8.20	1.614	6.07	1.180	5.30	0.948	5.83	1.154
	May	10.54	14.50	2.088	14.53	2.860	9.63	1.895	7.33	1.425	6.07	1.085	5.97	1.182
	June	11.26	16.56	2.385	15.63	3.076	9.67	1.903	8.30	1.614	5.80	1.037	6.27	1.241
	July	10.82	12.56	1.809	15.52	3.054	9.78	1.923	8.43	1.639	6.02	1.076	6.64	1.315
	Aug.	8.16	6.72	0.968	9.64	1.879	8.47	1.423	7.22	1.404	6.33	1.132	6.77	1.340

Table 1. Soil moisture data at Klondike Narrows Station (continued)

Year	Months	total	1st. foot		2nd. foot		3rd. foot		4th. foot		5th. foot		6th. foot	
		d"	W%	d"	W%	d"	W%	d"	W%	d"	W%	d"	W%	d"
1958	Oct.	6.44	4.53	0.652	7.47	1.470	5.37	1.057	5.00	0.972	5.77	1.031	6.33	1.253
	Nov.	6.18	5.53	0.796	6.87	1.352	4.76	0.937	4.70	0.914	5.50	0.983	6.03	1.194
	Jan.	5.63	8.43	1.214	6.30	1.240	4.27	0.840	4.13	0.803	3.50	0.626	4.60	0.911
	Feb.	7.91	13.38	1.927	10.83	2.112	4.87	0.958	4.72	0.916	4.88	0.874	5.67	1.123
	May	9.80	13.70	1.973	13.77	2.710	8.60	1.692	6.95	1.351	5.37	0.960	5.60	1.109
	June	11.05	15.43	2.222	15.40	3.031	9.93	1.954	7.80	1.516	6.13	1.096	6.23	1.234
	July	9.74	12.00	1.728	14.03	2.761	8.04	1.528	6.85	1.332	6.16	1.103	6.22	1.232
	Aug.	6.51	3.59	0.517	8.64	1.700	5.72	1.126	4.49	0.873	5.80	1.037	6.35	1.257
	Sept.	5.81	3.57	0.514	7.52	1.480	4.50	0.886	4.07	0.791	5.07	0.907	6.20	1.227

Table 1. Soil moisture data at Klondike Narrows Station (continued)

Year	Months	total d"	1st. foot W%	1st. foot d"	2nd. foot W%	2nd. foot d"	3rd. foot W%	3rd. foot d"	4th. foot W%	4th. foot d"	5th. foot W%	5th. foot d"	6th. foot W%	6th. foot d"
1959	Oct.	5.24	3.63	0.524	6.75	1.328	3.90	0.768	4.09	0.793	4.60	0.822	5.05	1.000
	Nov.	5.77	7.70	1.109	7.69	1.513	3.68	0.724	3.60	0.700	4.30	0.769	4.82	0.954
	Dec.	7.70	12.75	1.836	12.34	2.428	5.02	0.988	4.32	0.840	4.40	0.787	4.13	0.818
	Jan.	8.51	12.03	1.732	13.23	2.604	7.73	1.521	5.20	1.010	3.67	0.656	4.97	0.984
	Feb.	9.01	12.70	1.829	13.70	2.696	8.93	1.757	5.40	1.050	4.30	0.769	4.57	0.905
	Mar.	9.33	12.73	1.833	14.13	2.781	8.80	1.732	6.53	1.269	4.57	0.817	4.53	0.897
	Apr.	9.54	12.67	1.824	13.60	2.676	8.70	1.712	6.43	1.250	5.50	0.983	5.53	1.095
	May	9.64	13.24	1.907	13.67	2.690	8.55	1.683	6.73	1.308	5.04	0.901	5.80	1.148
	June	10.68	14.59	2.101	15.02	2.956	9.52	1.874	7.40	1.439	5.99	1.071	6.27	1.241
	July	8.11	9.43	1.358	11.10	2.184	5.83	1.147	6.10	1.186	5.50	0.983	6.30	1.247
	Aug.	3.80	3.34	0.480	3.85	0.758	2.64	0.519	3.10	0.602	3.48	0.622	4.12	0.816
	Sept.	2.84	2.93	0.422	2.97	0.584	2.20	0.433	2.04	0.396	1.79	0.320	3.45	0.683

Table 2. Streamflow, snow, and soil moisture data used in deriving prediction equation

Years	Months	Logan R. streamflow (acre-ft.)	Franklin Basin snow course (in.)	Garden City Summit snow course (in.)	Klondike * Narrows soil moist. (in.)
1956	Oct.	7,370			
	Nov.	6,490			
	Dec.	8,860			
	Jan.	8,010			
	Feb.	6,300			
	Mar.	8,200			
	Apr.	22,630			
	May	49,230			
	June	41,860			
	July	18,920			
	Aug.	12,560			
	Sept.	9,620			
1957	Oct.	8,870			4.46
	Nov.	7,640			4.47
	Dec.	7,130			4.63
	Jan.	6,530			5.01
	Feb.	5,990			5.23
	Mar.	7,230			5.23
	Apr.	11,020	31.6	21.3	9.55
	May	33,790			10.54
	June	49,690			11.26
	July	23,570			10.82
	Aug.	13,720			8.16
	Sept.	10,410			5.27
1958	Oct.	9,740			6.44
	Nov.	8,260			6.18
	Dec.	7,630			5.34
	Jan.	6,730			5.63
	Feb.	6,210			7.91
	Mar.	6,990			5.16
	Apr.	12,800	31.6	22.8	5.93
	May	47,200			9.80
	June	38,940			11.05
	July	17,630			9.74
	Aug.	12,370			6.51
	Sept.	9,810			5.81
1959	Oct.	8,580			5.24
	Nov.	7,690			5.77
	Dec.	7,060			7.70
	Jan.	6,270			8.51
	Feb.	5,560			9.01
	Mar.	6,510			9.33
	Apr.	12,600	24.9	16.0	9.54
	May	25,700			9.64

* This column was obtained from table 1.

Table 2. Streamflow, snow, and soil moisture data used in deriving prediction equation (continued)

Years	Months	Logan R. streamflow (acre-ft.)	Franklin Basin snow course (in.)	Garden City Summit snow course (in.)	Klondike Narrows soil moist. (in.)
1959	June	29,940			10.68
	July	14,910			8.11
	Aug.	10,420			3.80
	Sept.	8,400			2.84

watershed under Western Regional Research Project W-32 (Utah Project 459*). In this thesis only the Klondike Narrows station soil moisture data were used. Gypsum moisture blocks and fiberglass units were used for this purpose. Disturbed soil samples were obtained from each station hole and from one through six foot depths where the blocks and units were installed for purposes of calibrating the blocks and units.

Soil moisture measurements

Calibration of curves: Laboratory study (1) is to plot the calibration curves (13, 7) to obtain the relationship between the electrical resistance readings of gypsum blocks and fiberglass units, and the corresponding soil moisture content in percentage. These relationships as developed in the laboratory were utilized along with electrical resistance readings made in the field to estimate the volume of water in the 6-foot soil column (tables 1 and 2). The procedures for calibrating the blocks and units are as follows: Add sufficient water to the soil sample to bring the total water content to a desired value. Mix the entire soil sample thoroughly, until it is uniform in water content. Put the sample into a small plastic bag, a gypsum block and a fiberglass unit were then inserted in the soil sample. After the block and unit were installed, the soil compacted by hand so that a close contact was assured between the block and soil. Daily readings of resistance and temperature were made for each block and units, only one set of readings was taken each day in order to allow a sufficient amount of time for the moisture in the soil to reach a

* A cooperative project between the Utah Agricultural Experiment Station, the twelve western states associated in W-32, and the Civil and Irrigation Engineering Department of the Utah State University.

stage of equilibrium throughout the soil mass. After an equilibrium, as indicated by a constant resistance reading that was obtained, the readings of resistance and temperature were recorded. The block and unit were removed and the soil sample was quickly placed into a tared container and weighed. Then the soil sample was placed in a drying oven subjected to temperature of 110° C for about 24 hours. Loss in weight upon oven drying gave the moisture content of the sample. The percentage of soil moisture content may be expressed by the following equation:

$$w = \frac{(W_c + S_w) - (W_c + S_d)}{(W_c + S_d) - W_c} \quad \ldots \ldots \ldots \ldots \ldots \ldots \ldots (1)$$

in which,

w = soil moisture content in % by weight

W_c = weight of container

S_w = wet weight of soil

S_d = dry weight of soil

This procedure was repeated several times as described above, with different water contents. A curve was prepared for each gypsum block and fiberglass unit, showing resistance measured (bridge reading) and the corresponding soil moisture content in percentage (dry-weight basic). Thus the shape of the calibration curve was determined. The curves for each foot of depth and each hole are shown in figures 1 to 18.

Soil moisture determination: From each reading taken from the soil moisture measurement station, the corresponding soil moisture content can be determined from the calibration curves. Then, the depth of water in each foot of soil at the soil moisture measurement station can be determined by the following equation (9):

$$d = w\, A_s\, D \; . \quad (2)$$

where,

d = the depth of water stored in D depth of soil (Inches)

w = the percentage of soil moisture content

A_s = the apparent specific gravity of soil

D = depth of soil = 12 inches

Determination of best data for streamflow forecasting

Basic data should be complete and accurate. A careful study was made to determine whether any data were missing, how to replace missing data, and whether or not any changes were made in the location of measuring stations or procedures for making measurements which would have an influence on the data.

Streamflow data: The streamflow data of Logan River is complete for the study period. These are actual measured runoff throughout. There was no necessity to supply missing data by any indirect means. Data from these records were used in linear regression and correlation studies to determine which data will be best for streamflow forecasting (tables 3 to 7). Other studies were made, but only tables 3 to 7 are reported herein.

Snow survey data: Franklin Basin snow course (from 1924-1959) and Garden City Summit snow course (from 1931-1959) have continuous data for the study years. The history of the Franklin Basin snow course shows that this course did not change its location since it was established. Minor changes were made, however, in the number of samples collected at this station. In 1947 the Garden City Summit snow course was moved 1/2 mile north to its present location to eliminate excessive snow drifting. Linear regression and correlation studies were made; the results are listed in tables 3 and 4 with high degree of correlation. Since correlations are high, no adjustment in data were made to account for the change in location of the station.

Soil moisture data: Soil moisture data for this thesis were taken directly from the Klondike Narrows soil moisture measurement station. Its linear correlation coefficient showed a fairly good degree of significance (table 5), and it was of more significance than temperature and precipitation as shown in table 6 and table 7. Therefore, these data are used as a factor in the prediction equation.

Missing soil moisture data for the study years were replaced by linear regression equation (bottom table 5).

The mathematical model for streamflow forecasting

Fourier series: Fourier series may be expressed as,

$$f(x) = \frac{Ao}{2} + \sum_{1}^{\infty} (A_n \cos \frac{n\pi x}{c} + B_n \sin \frac{n\pi x}{c}) \quad . . . . (3)$$

where A_n and B_n are Fourier coefficients defined as follows:

$$A_n = \frac{1}{c} \int_{-c}^{c} f(x) \cos \frac{n\pi x}{c} dx \quad . . . . (4)$$

$$B_n = \frac{1}{c} \int_{-c}^{c} f(x) \sin \frac{n\pi x}{c} dx \quad . . . . (5)$$

A_o is the mean value of the function in the interval $- c = x = c$. The $f(x)$ can have only a finite number of finite discontinuities and maxima and minima over the interval c. In this study c is six, since the time interval involved is 12 months.

For purposes of this analysis equation (3) simply states that, if

Table 3. The linear regression equation and linear correlation coefficient of Franklin Basin snow course versus Logan River streamflow
(April 1 snow data versus April-Sept. streamflow)

No. of observation	Years	X : Franklin Basin snow course (in.)	Y : Logan R. streamflow (acre-ft.)	$x = X - \bar{X}$	$y = Y - \bar{Y}$
1	1924	25.1	115,540	- 1.78	-11,486
2	1925	28.3	129,800	1.42	2,744
3	1926	18.4	91,880	- 8.48	-35,146
4	1927	33.8	148,700	6.92	21,674
5	1928	30.0	146,300	3.12	19,274
6	1929	31.1	135,900	4.22	8,874
7	1930	26.8	99,380	- 0.08	-27,646
8	1931	14.6	55,060	-11.98	-71,966
9	1932	38.6	186,300	11.72	59,274
10	1933	28.2	123,840	1.32	- 3,186
11	1934	12.6	50,710	-14.28	-76,316
12	1935	24.4	114,350	- 2.48	-12,676
13	1936	39.7	119,950	12.82	72,924
14	1937	20.8	119,350	- 6.08	- 7,666
15	1938	24.9	147,070	- 1.98	20,043
16	1939	20.4	92,500	- 6.48	-34,526
17	1940	21.8	76,960	- 5.08	-50,066
18	1941	15.6	66,860	-11.28	-60,166
19	1942	17.8	89,810	- 9.08	-37,216
20	1943	38.8	173,870	11.92	46,844
21	1944	20.2	96,900	- 6.68	-30,126
22	1945	19.8	123,010	- 7.08	- 4,016
23	1946	30.2	168,450	3.32	41,424
24	1947	23.3	126,160	- 3.58	- 866
25	1948	26.5	152,340	- 0.38	25,314
26	1949	30.5	136,850	3.62	9,824
27	1950	41.3	213,850	14.42	86,824
28	1951	32.8	178,470	5.92	51,444
29	1952	40.2	168,620	13.32	41,594
30	1953	23.7	120,830	- 3.18	- 6,196
31	1954	23.3	86,250	- 3.58	-40,776
32	1955	23.8	99,310	- 3.08	-27,716
33	1956	31.8	154,820	4.92	27,794
34	1957	31.6	142,200	4.72	15,174
35	1958	31.6	138,770	4.72	11,744
36	1959	24.9	101,970	- 1.98	-25,056
Total		967.5	4,572,940	$x^2 = 1,927.37$	
Mean		$\bar{X} = 26.88$	$\bar{Y} = 127,026$		$y^2 = 53,949,746,011$

Table 3. (continued)

The linear regression equation:

$$\hat{Y} = bX + a$$

$$b = \frac{\Sigma xy}{\Sigma x^2} = \frac{9{,}312{,}202.46}{1{,}927.37} = 4{,}831.5$$

$$a = \bar{Y} - b\bar{X} = -2{,}844.72$$

$$\hat{Y} = 4{,}831.5X - 2{,}844.72$$

The linear correlation coefficient:

$$\Sigma \hat{y}^2 = \frac{(\Sigma xy)^2}{\Sigma x^2} = 44{,}992{,}458{,}450$$

$$r^2 = \frac{\Sigma \hat{y}^2}{\Sigma y^2} = 0.834$$

$$r = 0.9132 = 91.32\%$$

Table 4. The linear regression equation and linear correlation coefficient of Garden City Summit snow course versus Logan River streamflow:
(April 1 snow data versus April-Sept. streamflow)

No. of observation	Years	X: Garden City Summit snow course (in.)	Y: Logan R. streamflow (acre-ft.)	$x = X - \bar{X}$	$y = Y - \bar{Y}$
1	1931	4.4	55,060	-14.55	-72,714
2	1932	24.6	186,300	5.65	58,526
3	1933	18.2	123,840	0.75	- 3,934
4	1934	4.3	50,710	-14.65	-77,064
5	1935	17.9	114,350	- 1.05	-13,424
6	1936	33.9	199,950	14.95	72,176
7	1937	18.5	119,360	- 0.45	- 8,414
8	1938	20.4	147,070	1.45	19,296
9	1939	16.7	92,500	- 2.25	-35,274
10	1940	12.6	76,960	- 6.35	-50,814
11	1941	12.8	66,860	- 6.15	-60,914
12	1942	14.1	89,810	- 4.85	-37,964
13	1943	30.5	173,870	11.55	46,096
14	1944	15.6	96,900	- 3.35	-30,874
15	1945	15.5	123,010	- 3.45	- 4,764
16	1946	26.1	168,450	7.15	40,676
17	1947	16.0	126,160	- 2.95	- 1,614
18	1948	15.4	152,340	- 3.55	24,566
19	1949	22.8	136,850	3.85	9,076
20	1950	28.9	213,850	9.95	86,076
21	1951	24.4	178,470	5.45	50,696
22	1952	26.6	168,620	7.65	40,846
23	1953	14.5	120,830	- 4.45	- 6,944
24	1954	19.5	86,250	0.55	-41,524
25	1955	14.6	99,310	- 4.35	-28,464
26	1956	20.7	154,820	1.75	27,046
27	1957	21.3	142,200	2.35	14,426
28	1958	22.8	138,770	3.85	10,996
29	1959	16.0	101,970	- 2.95	-25,804
Total		549.6	63,705,440	x^2 = 1.294.59	y^2 = 50,878,080,884
Mean		$\bar{X}$ = 18.95	$\bar{Y}$ = 147,774		

The linear regression equation:

$$\hat{Y} = bX + a$$

$$b = \frac{\Sigma xy}{\Sigma x^2} \quad \frac{7,278,663.3}{1,294.59} = 5,622.37$$

$$a = \bar{Y} - b\bar{X} = 21,230$$

Table 4. (continued)

$$\hat{Y} = 5{,}622.37X + 21{,}230$$

The linear correlation coefficient:

$$\Sigma \hat{y}^2 = \frac{(\Sigma xy)^2}{\Sigma x^2} = 40{,}923{,}334{,}360$$

$$r^2 = \frac{\Sigma \hat{y}^2}{\Sigma y^2} = 0.8043$$

$$r = 0.8968 = 89.68\%$$

Table 5. The linear regression equation and linear correlation coefficient of Klondike Narrows soil moisture versus Logan River streamflow:

No. of observation	Year	Months	X: Soil moisture (in.)	Y: Logan R. streamflow (acre-ft.)	$x = X - \bar{X}$	$y = Y - \bar{Y}$
1	1957	Oct.	4.46	8,870	-3.07	- 6,243
2		Nov.	4.47	7,640	-3.06	- 7,473
3		Dec.	4.63	7,130	-2.90	- 7,983
4		Jan.	5.01	6,530	-2.52	- 8,583
5		Feb.	5.23	5,990	-2.30	- 9,123
6		Apr.	9.55	11,020	2.02	- 4,093
7		May	10.54	33,790	3.01	18,677
8		June	11.26	49,690	3.73	34,577
9		July	10.82	23,570	3.29	8,457
10		Aug.	8.16	13,720	0.63	- 1,393
11	1958	Oct.	6.44	9,740	-1.09	- 5,373
12		Nov.	6.18	8,260	-1.35	- 6,853
13		Jan.	5.63	6,730	-1.90	- 8,383
14		Feb.	7.91	6,210	0.38	- 8,903
15		May	9.80	47,220	2.27	32,107
16		June	11.05	38,940	3.52	23,827
17		July	9.74	17,630	2.21	2,517
18		Aug.	6.51	12,370	-1.02	- 2,743
19		Sept.	5.81	9,810	-1.72	- 5,303
20	1959	Oct.	5.24	8,580	-2.29	- 6,533
21		Nov.	5.77	7,690	-1.76	- 7,423
22		Dec.	7.70	7,060	0.17	- 8,053
23		Jan.	8.51	6,270	0.98	- 8,843
24		Feb.	9.01	5,560	1.48	- 9,553
25		March	9.33	6,510	1.80	- 8,603
26		Apr.	9.54	12,600	2.01	- 2,513
27		May	9.64	25,700	2.11	10,587
28		June	10.68	29,940	3.15	14,827
29		July	8.11	14,910	0.58	- 203
30		Aug.	3.80	10,420	-3.73	- 4,693
31		Sept.	2.84	8,400	-4.69	- 6,713
Total			233.37	468,500	x^2 = 179.73	y^2 = 4,627,468,039
Mean			$\bar{X}$ = 7.53	$\bar{Y}$ = 15,112.9		

The linear regression equation:

$$\hat{Y} = bX + a$$

$$b = \frac{\Sigma xy}{\Sigma x^2} = \frac{614,782.08}{179.73} = 3,420.59$$

$$a = \bar{Y} - b\bar{X} = -10,644$$

Table 5. (continued)

$$\hat{Y} = 3,420.59X - 10,644$$

The linear correlation coefficient:

$$\Sigma\hat{y}^2 = \frac{(\Sigma xy)^2}{\Sigma x^2} = 2,102,915,517$$

$$r^2 = \frac{\Sigma\hat{y}^2}{\Sigma y^2} = 0.4544$$

$$r = 0.6741 = 67.41\%$$

Table 6. The linear regression equation and linear correlation coefficient of Logan temperature versus Logan River streamflow:

No. of observation	Year	Months	X: Temperature (F^o)	Y: Logan R. streamflow (acre-ft.)	$x = X - \bar{X}$	$y = Y - \bar{Y}$
1	1957	Oct.	50.3	8,870	1.57	- 5,396
2		Nov.	30.8	7,640	-17.93	- 6,626
3		Dec.	25.7	7,130	-23.03	- 7,136
4		Jan.	20.2	6,530	-28.53	- 7,736
5		Feb.	33.6	5,990	-15.13	- 8,276
6		March	40.3	7,230	- 8.43	- 7,036
7		Apr.	45.2	11,020	- 3.53	- 3,246
8		May	55.7	33,790	6.97	19,524
9		June	64.2	49,690	15.47	35,424
10		July	72.6	23,570	23.87	9,304
11		Aug.	72.4	13,720	23.67	- 546
12		Sept.	61.6	10,410	12.87	- 3,856
13	1958	Oct.	49.6	9,740	0.87	- 4,526
14		Nov.	31.6	8,260	-17.13	- 6,006
15		Dec.	31.5	7,630	-17.23	- 6,636
16		Jan.	22.4	6,730	-26.33	- 7,536
17		Feb.	36.4	6,210	-12.33	- 8,056
18		March	35.7	6,990	-13.03	- 7,276
19		Apr.	44.3	12,800	- 4.43	- 1,466
20		May	62.3	47,220	13.57	32,954
21		June	67.1	38,940	18.37	24,674
22		July	71.3	17,630	22.57	3,364
23		Aug.	74.4	12,370	25.67	- 1,896
24		Sept.	62.3	9,810	13.57	- 4,456
25	1959	Oct.	53.2	8,580	4.47	- 5,686
26		Nov.	35.3	7,690	-13.43	- 6,576
27		Dec.	34.3	7,060	-14.43	- 7,206
28		Jan.	30.2	6,270	-18.53	- 7,996
29		Feb.	30.9	5,560	-17.83	- 8,706
30		March	38.1	6,510	-10.63	- 7,756
31		Apr.	48.5	12,600	- 0.23	- 1,666
32		May	53.2	25,700	4.47	11,434
33		June	67.4	29,940	18.67	15,674
34		July	72.7	14,910	23.97	644
35		Aug.	70.2	10,420	21.47	- 3,846
36		Sept.	58.9	8,400	10.17	- 5,866
Total			1,754.4	513,560	x^2 = 9,754.58	y^2 = 4,813,202,496
Mean			$\bar{X}$ = 48.73	$\bar{Y}$ = 14,266		

Table 6. (continued)

The linear regression equation:

$$\hat{Y} = bX + a$$

$$b = \frac{\Sigma xy}{\Sigma x^2} = \frac{3,784,624}{9,754.58} = 387.98$$

$$a = \bar{Y} - b\bar{X} = -4,640$$

$$\hat{Y} = 387.98X - 4,640$$

The linear correlation coefficient:

$$\Sigma \hat{y}^2 = \frac{(\Sigma xy)^2}{\Sigma x^2} = 1,468,374,730$$

$$r^2 = \frac{\Sigma \hat{y}^2}{\Sigma y^2} = 0.3051$$

$$r = 0.5524 = 55.24\%$$

Table 7. The linear regression equation and linear correlation coefficient of Logan precipitation versus Logan River streamflow:

No. of observation	Year	Months	X: Precipitation (in.)	Y: Logan R. streamflow (acre-ft.)	$x = X - \bar{X}$	$y = Y - \bar{Y}$
1	1957	Oct.	1.12	8,870	-0.254	- 5,396
2		Nov.	1.17	7,640	-0.204	- 6,626
3		Dec.	1.39	7,130	0.016	- 7,136
4		Jan.	1.65	6,530	0.276	- 7,736
5		Feb.	1.76	5,990	0.386	- 8,276
6		March	2.00	7,230	0.626	- 7,036
7		Apr.	3.41	11,020	2.036	- 3,246
8		May	3.02	33,790	1.646	19,524
9		June	1.29	49,690	-0.084	35,424
10		July	0.08	23,570	-1.294	9,304
11		Aug.	0.50	13,720	-0.874	- 546
12		Sept.	0.50	10,410	-0.874	- 3,856
13	1958	Oct.	0.88	9,740	-0.494	- 4,526
14		Nov.	1.27	8,260	-0.104	- 6,006
15		Dec.	1.44	7,630	0.066	- 6,636
16		Jan.	1.07	6,730	-0.304	- 7,536
17		Feb.	1.40	6,210	0.026	- 8,056
18		March	2.61	6,990	1.236	- 7,276
19		Apr.	0.77	12,800	-0.604	- 1,466
20		May	0.85	47,220	-0.524	32,954
21		June	0.41	38,940	-0.964	24,674
22		July	0.53	17,630	-0.844	3,364
23		Aug.	0.69	12,370	-0.684	- 1,896
24		Sept.	0.43	9,810	-0.944	- 4,456
25	1959	Oct.	0.02	8,580	-1.357	- 5,686
26		Nov.	2.97	7,690	1.596	- 6,576
27		Dec.	1.60	7,060	0.226	- 7,206
28		Jan.	1.59	6,270	0.216	- 7,996
29		Feb.	1.73	5,560	0.356	- 8,706
30		March	1.30	6,510	-0.074	- 7,756
31		Apr.	2.48	12,600	1.106	- 1,666
32		May	1.70	25,700	0.326	11,434
33		June	1.28	29,940	-0.094	15,674
34		July	0.08	14,910	-1.294	644
35		Aug.	2.38	10,420	1.006	- 3,846
36		Sept.	2.10	8,400	0.726	- 5,866
Total			49.47	513,560	x^2 = 25.480	y^2 = 4,813,202,496
Mean			$\bar{X}$ = 1.374	$\bar{Y}$ = 14,266		

Table 7. (continued)

The linear regression equation:

$$\hat{Y} = bX + a$$

$$b = \frac{\Sigma xy}{\Sigma x^2} = \frac{-51,969.4}{25.48} = -3,961.5$$

$$a = \bar{Y} - b\bar{X} = 19,709$$

$$\hat{Y} = -3961.5X - 19,709$$

The linear correlation coefficient:

$$\Sigma \hat{y}^2 = \frac{(\Sigma xy)^2}{\Sigma x^2} = 105,997,587.7$$

$$r^2 = \frac{\Sigma \hat{y}^2}{\Sigma y^2} = 0.022$$

$$r = 0.1484 = 14.84\%$$

$f(x)$ represents actual soil moisture or streamflow data plotted on a time scale, we can fit a trigonometric curve of the form represented by the right hand side of the equation to these data by an appropriate selection of the coefficients $A_1, A_2, A_3, \ldots . A_n$ and $B_1, B_2, B_3, \ldots . B_n$. Procedure (14) for selection of these coefficients is indicated in equations (4) and (5). Examples of the numerical procedure will follow.

In this thesis equation (3) was simplified and utilized to represent monthly mean soil moisture and monthly streamflow in time as follows:

A. Soil moisture

$$Mm = \bar{M} + Am_1 \cos\theta + Am_2 \sin\theta + Am_3 \cos 2\theta + Am_4 \sin 2\theta \quad . \ . \ . \ (6)$$

where, Mm = Mean soil moisture for any month of the year in question.

$\bar{M}$ = Average soil moisture for 12 months (mean of monthly mean soil moistures).

Am_1, Am_2, Am_3, and Am_4 are Fourier coefficients determined from actual soil moisture data.

θ = Time (2π radians or 360 degree is one year).

Only four terms of the series were considered necessary to represent monthly mean soil moisture adequately. Theoretically, the accuracy of this method can be increased by adding additional terms to the Fourier series, however, the work of computation for finding the multiple linear regression coefficients is considerably increased.

B. Streamflow

$$Fm = \bar{F} + A_{F_1} \cos\theta + A_{F_2} \sin\theta + A_{F_3} \cos 2\theta + A_{F_4} \sin 2\theta \quad . \ . \ (7)$$

where, Fm = Mean streamflow for any month of the year in in question.

$\bar{F}$ = Average streamflow for 12 months (based on a water year).

A_{F_1}, A_{F_2}, A_{F_3}, and A_{F_4} are Fourier coefficients determined from actual streamflow data.*

θ = Time (2π radians or 360 degrees is one year).

The streamflow forecasting equation: The streamflow forecasting equation, using water content of the April 1 snow cover as indicated by the two snow courses, the average, and Fourier coefficients from equation (6) and (7) is as follows:

$$\hat{Y} = b_0 + b_1x_1 + b_2x_2 + b_3\bar{M} + b_4A_{m_1} + b_5A_{m_2} + b_6A_{m_3} + b_7A_{m_4}$$

$$+ b_8\bar{F} + b_9A_{F_1} + b_{10}A_{F_2} + b_{11}A_{F_3} + b_{12}A_{F_4} \quad \ldots\ldots\ldots\ldots \quad (8)$$

where, $\hat{Y}$ = The predicted monthly streamflow

b_o = A constant (Y-axis intercept).

b_1, b_2, ... b_{12} are the multiple linear regression coefficients.

x_1, and x_2 are the April 1 measurement of the water content of the snow courses (Franklin Basin and Garden City Summit).

$\bar{M}$ = Average soil moisture for 12 months (mean of monthly mean soil moisture).

A_{M_1}, A_{M_2}, A_{M_3}, and A_{M_4} are Fourier coefficients determined from actual soil moisture data.

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* Examples of the procedure for the numerical computation of the Fourier coefficients for soil moisture and streamflow data are illustrated in tables 8 and 9. See also equations (10) and (11).

Table 8. Example for computation of Fourier soil moisture coefficients 1957
(Soil moisture at Klondike Narrows, Utah)

Month	X^*	(A_{F_1})	Coeff. $\cos\theta$	(A_{F_2})	Coeff. $\sin\theta$	(A_{F_3})	Coeff.	(A_{F_4})	Coeff. $\sin 2\theta$
		$\cos\theta$	$x\cos\theta$	$\sin\theta$	$x\sin\theta$	$\cos 2\theta$	$x\cos 2\theta$	$\sin 2\theta$	$x\sin 2\theta$
Oct.	4.46	1.000	4.46	0.000	0	1.000	4.46	0.000	0
Nov.	4.47	0.866	3.87	0.500	2.24	0.500	2.24	0.866	3.87
Dec.	4.63	0.500	2.32	0.866	4.01	-0.500	-2.32	0.866	4.01
Jan.	5.01	0.000	0	1.000	5.01	-1.000	-5.01	0.000	0
Feb.	5.23	-0.500	-2.62	0.866	4.53	-0.500	-2.62	-0.866	-4.53
Mar.	5.23	-0.866	-4.53	0.500	2.62	0.500	2.62	-0.866	-4.53
Apr.	9.55	-1.000	-9.55	0.000	0	1.000	9.55	0.000	0
May	10.54	-0.866	-9.13	-0.500	-5.27	0.500	5.27	0.866	9.13
June	11.26	-0.500	-5.63	-0.866	-9.75	-0.500	-5.63	0.866	9.75
July	10.82	0.000	0	-1.000	-10.82	-1.000	-10.82	0.000	0
Aug.	8.16	0.500	4.08	-0.866	-7.07	-0.500	-4.08	-0.866	-7.07
Sept.	5.27	0.866	4.56	-0.500	-2.64	-0.500	2.64	-0.866	-4.56
Sum	84.63		-12.17		-17.14		-3.70		6.07
A_M	7.05		- 2.028		- 2.857		-0.617		1.012

* The X values are the current year monthly mean soil moisture beginning from Oct. 1956 to Sept. 1957.

Table 9. Example for computation of Fourier streamflow coefficients
1957
(Streamflow on Logan River, Utah)

Month	X^*	(A_{F_1})	Coeff. $\cos\theta$	(A_{F_2})	Coeff. $\sin\theta$	(A_{F_3})	Coeff. $\cos 2\theta$	(A_{F_4})	Coeff.
		$\cos\theta$	$x\cos\theta$	$\sin\theta$	$x\sin\theta$	$\cos 2\theta$	$x\cos 2\theta$	$\sin 2\theta$	$x\sin 2\theta$
Oct.	7,370	1.000	7,370	0.000	0	1.000	7,370	0.000	0
Nov.	6,490	0.866	5,620	0.500	3,245	0.500	3,245	0.866	5,620
Dec.	8,860	0.500	4,430	0.866	7,673	-0.500	-4,430	0.866	7,673
Jan.	8,010	0.000	0	1.000	8,010	-1.000	-8,010	0.000	0
Feb.	6,300	-0.500	-3,150	0.866	5,456	-0.500	-3,150	-0.866	-5,456
Mar.	8,200	-0.866	-7,101	0.500	4,100	0.500	4,100	-0.866	-7,101
Apr.	22,630	-1.000	-22,630	0.000	0	1.000	22,630	0.000	0
May	49,230	-0.866	-42,630	-0.500	-24,615	0.500	24,615	0.866	42,633
June	41,860	-0.500	-20,930	-0.866	-36,251	-0.500	-20,930	0.866	36,251
July	18,920	0.000	0	-1.000	-18,920	-1.000	-18,920	0.000	0
Aug.	12,560	0.500	6,280	-0.866	-10,877	-0.500	-6,280	-0.866	-10,877
Sept.	9,620	0.866	8,331	-0.500	-4,810	0.500	4,810	-0.866	-8,331
Sum	200,050		-64,413		-66,989		5,050		60,412
A_F	16,670		-10,736		-11,165		842		10,069

* The X values are the antecedent year monthly streamflow beginning from Oct. 1955 to Sept. 1956.

$\bar{F}$ = Average streamflow for 12 months (based on a water year).

A_{F_1}, A_{F_2}, A_{F_3}, and A_{F_4} are Fourier coefficients determined from actual streamflow data.

<u>Simultaneous equations for determination of the multiple linear regression coefficients</u> (12):

$$\left.\begin{array}{llllllll}
b_oN & +b_1\Sigma X_1 & +b_2\Sigma X_2 & +b_3\Sigma\bar{M} & \ldots + b_{11}\Sigma A_{F_3} & + b_{12}\Sigma A_{F_4} & = \Sigma Y \\
b_o\Sigma X_1 & +b_1\Sigma X_1^2 & +b_2\Sigma X_1X_2 & +b_3\Sigma X_1\bar{M} & \ldots + b_{11}\Sigma X_1A_{F_3} & + b_{12}\Sigma X_1A_{F_4} & = \Sigma X_1Y \\
b_o\Sigma X_2 & +b_1\Sigma X_2X_1 & +b_2\Sigma X_2^2 & +b_3\Sigma X_2\bar{M} & \ldots + b_{11}\Sigma X_2A_{F_3} & + b_{12}\Sigma X_2A_{F_4} & = \Sigma X_2Y \\
b_o\Sigma\bar{M} & +b_1\Sigma\bar{M}X_1 & +b_2\Sigma\bar{M}X_2 & +b_3\Sigma\bar{M}^2 & \ldots + b_{11}\Sigma\bar{M}A_{F_3} & + b_{12}\Sigma\bar{M}A_{F_4} & = \Sigma\bar{M}Y \\
b_o\Sigma A_{M_1} & +b_1\Sigma A_{M_1}X_1 & +b_2\Sigma A_{M_1}X_2 & +b_3\Sigma A_{M_1}\bar{M} & \ldots + b_{11}\Sigma A_{M_1}A_{F_3} & + b_{12}\Sigma A_{M_1}A_{F_4} & = \Sigma A_{M_1}Y \\
b_o\Sigma A_{M_2} & +b_1\Sigma A_{M_2}X_1 & +b_2\Sigma A_{M_2}X_2 & +b_3\Sigma A_{M_2}\bar{M} & \ldots + b_{11}\Sigma A_{M_2}A_{F_3} & + b_{12}\Sigma A_{M_2}A_{F_4} & = \Sigma A_{M_2}Y \\
b_o\Sigma A_{M_3} & +b_1\Sigma A_{M_3}X_1 & +b_2\Sigma A_{M_3}X_2 & +b_3\Sigma A_{M_3}\bar{M} & \ldots + b_{11}\Sigma A_{M_3}A_{F_3} & + b_{12}\Sigma A_{M_3}A_{F_4} & = \Sigma A_{M_3}Y \\
b_o\Sigma A_{M_4} & +b_1\Sigma A_{M_4}X_1 & +b_2\Sigma A_{M_4}X_2 & +b_3\Sigma A_{M_4}\bar{M} & \ldots + b_{11}\Sigma A_{M_4}A_{F_3} & + b_{12}\Sigma A_{M_4}A_{F_4} & = \Sigma A_{M_4}Y \\
b_o\Sigma\bar{F} & +b_1\Sigma\bar{F}X_1 & +b_2\Sigma\bar{F}X_2 & +b_3\Sigma\bar{F}\bar{M} & \ldots + b_{11}\Sigma\bar{F}A_{F_3} & + b_{12}\Sigma\bar{F}A_{F_4} & = \Sigma\bar{F}Y \\
b_o\Sigma A_{F_1} & +b_1\Sigma A_{F_1}X_1 & +b_2\Sigma A_{F_1}X_2 & +b_3\Sigma A_{F_1}\bar{M} & \ldots + b_{11}\Sigma A_{F_1}A_{F_3} & + b_{12}\Sigma A_{F_1}A_{F_4} & = \Sigma A_{F_1}Y \\
b_o\Sigma A_{F_2} & +b_1\Sigma A_{F_2}X_1 & +b_2\Sigma A_{F_2}X_2 & +b_3\Sigma A_{F_2}\bar{M} & \ldots + b_{11}\Sigma A_{F_2}A_{F_3} & + b_{12}\Sigma A_{F_2}A_{F_4} & = \Sigma A_{F_2}Y \\
b_o\Sigma A_{F_3} & +b_1\Sigma A_{F_3}X_1 & +b_2\Sigma A_{F_3}X_2 & +b_3\Sigma A_{F_3}\bar{M} & \ldots + b_{11}\Sigma A_{F_3}^2 & + b_{12}\Sigma A_{F_3}A_{F_4} & = \Sigma A_{F_3}Y \\
b_o\Sigma A_{F_4} & +b_1\Sigma A_{F_4}X_1 & +b_2\Sigma A_{F_4}X_2 & +b_3\Sigma A_{F_4}\bar{M} & \ldots + b_{11}\Sigma A_{F_4}A_{F_3} & + b_{12}\Sigma A_{F_4}^2 & = \Sigma A_{F_4}Y
\end{array}\right\} (9)$$

where, N = the number of years in this study.

Y = the measured monthly streamflow in question.

A separate set of regression coefficients and constants is computed for each month (tables 10 and 11).

For more detailed explanation of the meanings denoted by the symbols in the streamflow forecasting equation and the simultaneous equations, the following tabulation is included:

Snow courses (the April 1 water content of the snow)

X_1 = Franklin Basin

X_2 = Garden City Summit

Soil moistures (Klondike Narrows, Utah)

$\bar{M}$ = Average soil moisture for 12 months (mean of monthly mean soil moistures)

$$A_{M_1} = 1/6\ (\Sigma X_M \cos\theta)$$
$$A_{M_2} = 1/6\ (\Sigma X_M \sin\theta)$$
$$A_{M_3} = 1/6\ (\Sigma X_M \cos 2\theta) \qquad (10)$$
$$A_{M_4} = 1/6\ (\Sigma X_M \sin 2\theta)$$

where, X_M = Monthly mean soil moisture for the month in question.

Derivation of these equations is shown in reference (10) of the literature cited (see table 8 for details of computation).

Streamflow (Logan River, Utah)

$\bar{F}$ = Average streamflow for 12 months, in acre-feet (based on a water year).

$$A_{F_1} = 1/6\ (\Sigma X_F \cos\theta)$$
$$A_{F_2} = 1/6\ (\Sigma X_F \sin\theta)$$
$$A_{F_3} = 1/6\ (\Sigma X_F \cos 2\theta) \qquad (11)$$
$$A_{F_4} = 1/6\ (\Sigma X_F \sin 2\theta)$$

where, X_F = Monthly streamflow for the month in question (see table 9 for details of computation).

Table 10. The regression coefficients "b_n" for each month

Months	October (Y_1)	November (Y_2)	December (Y_3)	January (Y_4)	February (Y_5)	March (Y_6)
b_1	+ 97.30000	- 14.31000	-354.90000	+14.67400	- 34410.785	+40.68500
b_2	+ 40.76500	+ 19.15500	+ 13.56500	+10.91700	- 76004.350	+ 0.65500
b_3	+ 47.40000	+ 28.55000	+ 20.00000	+13.42000	- 90777.400	- 3.35000
b_4	-205.12000	- 37.26000	- 35.32000	-36.60600	+322631.340	- 1.58000
b_5	+ 44.51000	+ 32.29000	+ 25.36000	+11.20000	+ 174.170	- 9.41000
b_6	- 59.40000	+121.90000	+ 49.20000	+23.72000	+ 17124.800	+31.30000
b_7	- 6.70000	- 44.30000	- 5.60000	- 2.77000	- 4811.300	-16.70000
b_8	+ 0.41471	- 0.32922	- 0.08093	+ 0.04058	+ 80.392	+ 0.35307
b_9	+ 0.00117	- 0.00388	- 0.00199	- 0.00311	- 33.669	- 0.00453
b_{10}	+ 0.85925	- 0.64645	- 0.06926	- 0.13610	- 178.943	+ 0.27787
b_{11}	- 0.65823	+ 0.01900	- 0.23043	- 0.10166	+ 66.295	- 0.32289
b_{12}	- 0.00630	+ 0.44838	+ 0.47658	+ 0.24906	- 22.122	+ 0.53736

Table 10. The regression coefficients "b_n" for each month (continued)

Months	April (Y_7)	May (Y_8)	June (Y_9)	July (Y_{10})	August (Y_{11})	September (Y_{12})
b_1	+17410308.27	+1565.9000	+1067.2000	+1547.0000	+ 53190.84	+113.55500
b_2	+ 2385195.90	+ 792.6000	- 156.1500	+ 170.4000	- 56404.76	+ 4.45000
b_3	+ 1390475.70	+ 881.0000	- 326.5000	- 5.5000	- 84464.55	- 5.60000
b_4	-22606585.64	-3919.0000	+ 562.6000	-2388.6000	+1448845.02	- 13.28000
b_5	+ 123239.78	+ 711.9000	- 472.3000	- 287.2000	- 79105.63	- 22.71000
b_6	-10860885.10	- 594.0000	+ 920.0000	-4742.0000	+ 306705.00	+ 89.80000
b_7	-10491268.50	- 394.0000	- 676.0000	+1036.0000	-1354317.20	- 46.40000
b_8	+ 125671.87	+ 7.3893	+ 0.4637	+ 18.3672	+ 790.92	+ 0.96447
b_9	+ 1810.68	- 0.0277	- 0.1002	+ 0.0893	+ 34.41	- 0.01300
b_{10}	+ 354789.64	+ 8.8546	+ 9.1397	+ 38.5000	+ 3980.94	+ 0.71797
b_{11}	- 143466.09	- 7.9968	- 7.7989	- 15.3164	- 785.84	- 0.89775
b_{12}	- 60180.68	- 3.6366	+ 12.4886	- 7.2340	- 1415.78	+ 1.51353

Table 11. The regression coefficients "b_o" for each month

Months	October (Y_1)	November (Y_2)	December (Y_3)	January (Y_4)
b_o	+7,575.324	+1,337.695	+13,076.156	+1,198.867

Months	February (Y_5)	March (Y_6)	April (Y_7)	May (Y_8)
b_o	+487,381.397	-2,021.127	+1,956,365,038.032	-26,153.330

Months	June (Y_9)	July (Y_{10})	August (Y_{11})	September (Y_{12})
b_o	-11,651.977	+169,769.742	+50,927,370.000	-15,937.000

Computation and presentation

All of the computations for this thesis were made on a desk electric calculator. Each step of the computations were checked to eliminate errors. Data used in this example are listed in table 2. Actual measured data for the October-March, as well as the April-September periods, were used. In forecasting streamflow, data are not available for the April-September period. The long-time monthly means of soil moisture data and streamflow data are used for prediction. The general system of linear normal equations is shown in equation (9). The data for each variable are available for three years (1957-1959). The sums, sums of squares, and sums of products were computed from the data in table 12. (For example: In table 13, $\Sigma X_1X_2 = 1{,}791.96$ in column 3, row 2 was computed by adding the products of X_1 and X_2 in table 12, and $\Sigma X_2^2 = 1{,}229.53$ in column 3, row 3 was obtained by summing the squares of X_2 in table 12). The results of these computations are shown in table 13, which is designated "The uncorrected original information matrix," and the corrected original information matrix is shown in table 14. The portion below the main diagonal is omitted from the table because of symmetry.

To facilitate the inversion of the matrix, the original information matrix is coded. Table 15 shows this matrix in coded form. The coding factor notation is K_r (for rows) and K_c (for columns), as shown in table 14.

The inverse matrix and regression coefficients were obtained by a "Machine Method of Matrix Inversion," developed by Dr. Rex L. Hurst. The coded inverse matrix forward solution is in table 16. The coded inverse matrix back solution and coded multiple linear regression coefficients

Table 12. Tabulation of soil moisture Fourier coefficients, streamflow Fourier coefficients, and snow water content

April 1 water content of snow at 2 snow course (in.)
(Franklin Basin X_1, and Garden City Summit X_2)

Years	X_1	X_2
1957	31.6	21.3
1958	31.6	22.8
1959	24.9	16.0

Klondike Narrows annual monthly mean soil moisture and Fourier coefficients.

Years	$\bar{M}$ (in.)	A_{M_1}	A_{M_2}	A_{M_3}	A_{M_4}
1957	7.05	-2.028	-2.857	-0.617	1.012
1958	7.13	-0.938	-1.665	-0.823	1.007
1959	7.51	-2.895	0.608	-0.607	1.273

Logan River annual monthly mean streamflow and Fourier coefficients

Years	$\bar{F}$ (acre-ft.)	A_{F_1}	A_{F_2}	A_{F_3}	A_{F_4}
1957	16,700	-10,736	-11,165	842	10,069
1958	15,500	-6,578	-12,543	-3,157	8,790
1959	15,400	-7,819	-10,705	289	9,620

Table 13. Uncorrected original information matrix

Row \ Col.	1	2	3	4	5	6	7	8	9	10	11	12	13
1	3	88.1	60.10	21.8	-5.9	-3.9	-2.0	3.3	47600	-25133	-34413	-2026	28479
2		2617.1	1791.96	635.1	-165.8	-127.8	-60.6	95.5	1400980	-741815	-1015727	-65958	835482
3			1229.53	432.9	-110.9	-89.1	-41.6	64.9	955510	-503759	-695075	-49421	568802
4				156.9	-42.7	-27.4	-14.8	23.9	343904	-181311	-248539	-14403	205905
5					13.4	5.6	3.8	-6.7	-92990	50579	65399	417	-56515
6						10.6	2.8	-3.8	-64156	36817	46274	3027	-37554
7							1.4	-2.2	-32408	16784	23710	1903	-19286
8								3.7	52113	-27442	-37557	-1959	31288
9									756300000	-401662800	-545729000	-30421500	452545300
10										219668541	286077689	9467343	-241140184
11											396581099	27103576	-325655455
12												10759134	-16491752
13													271193261

Table 14. Corrected original information matrix

Code factor	$K_{c1}=2^{-1}$	$K_{c2}=2^{-1}$	$K_{c3}=5$	$K_{c4}=2$	$K_{c5}=1$	$K_{c6}=10$	$K_{c7}=10$	$K_{c8}=10^{-3}$	$K_{c9}=10^{-3}$	$K_{c10}=10^{-3}$	$K_{c11}=10^{-3}$	$K_{c12}=10^{-3}$
$K_{r1}=2^{-1}$	29.9	27.0	-1.88	6.31	-12.82	-0.505	-1.18	3126.7	-3743.1	-5132	-6461	-850.9
$K_{r2}=2^{-1}$		25.5	-1.63	6.51	-10.68	-0.610	-1.07	1923.3	-261.4	-5668	-8833	-1727.6
$K_{r3}=5$			0.12	-0.35	0.85	0.023	0.07	-244.0	400.9	267	245	2.2
$K_{r4}=2$				1.92	-2.05	-0.219	-0.25	4.9	1477.3	-1832	-3541	-876.4
$K_{r5}=1$					5.46	0.093	0.50	-2054.1	4026.9	1376	383	-397.9
$K_{r6}=10$						0.030	0.02	70.9	-365.1	228	521	146.1
$K_{r7}=10$							0.05	-119.9	136.8	205	264	36.7
$K_{r8}=10^{-3}$								1046667.0	-2885867.0	290600	1724366	678500.0
$K_{r9}=10^{-3}$									9112645.0	-2222954	-7505810	-2552615.0
$K_{r10}=10^{-3}$										1829576	3863330	1027154.0
$K_{r11}=10^{-3}$											9390909	2741066.0
$K_{r12}=10^{-3}$												842114.0

Table 15. Coded information matrix (See original information matrix and code factors in table 12.)

a_1	a_2	a_3	a_4	a_5	a_6	a_7	a_8	a_9	a_{10}	a_{11}	a_{12}	J
7.482	6.756	-4.690	6.307	-6.407	-2.525	-5.885	1.563	-1.871	-2.566	-3.231	-0.425	-5.493
	6.382	-4.077	6.513	-5.339	-3.050	-5.330	0.962	-0.131	-2.834	-4.417	-0.864	-5.429
		3.025	-3.510	4.255	1.150	3.650	-1.220	2.005	1.333	1.225	0.011	3.156
			7.693	-4.102	-4.373	-5.015	0.010	2.954	-3.665	-7.082	-1.753	-6.023
				5.458	0.934	5.010	-2.054	4.027	1.376	0.383	-0.398	3.143
					2.973	2.036	0.709	-3.651	2.285	5.209	1.461	3.157
						4.630	-1.199	1.368	2.053	2.641	0.367	4.325
							1.047	-2.886	0.291	1.724	0.678	-0.376
								9.113	-2.223	-7.506	-2.553	-1.354
									1.830	3.863	1.027	2.770
										9.391	2.741	4.943
											0.842	1.135

Table 16. The coded inverse matrix (Forward solution)

A_1	A_2	A_3	A_4	A_5	A_6	A_7	A_8	A_9	A_{10}	A_{11}	A_{12}	J
7.48175	6.75600	-4.69000	6.30700	-6.40750	-2.52500	-5.88500	1.65333	-1.87153	-2.56610	-3.23051	-0.42545	-5.49301
1.00000	0.90299	-0.62685	0.84298	-0.85641	-0.33748	-0.78658	0.20895	-0.25014	-0.34298	-0.43178	-0.05686	-0.73416
	0.28115	0.15749	0.81783	0.44690	-0.76999	-0.01587	-0.45000	1.55923	-0.51673	-1.49962	-0.47966	-0.46925
	1.00000	0.56016	2.90887	1.58954	-2.73871	-0.05644	-1.60056	5.54590	-1.83791	-5.33387	-1.70606	-1.66903
		-0.00314	-0.01454	-0.01190	-0.00146	-0.03017	0.01204	-0.04178	0.01388	0.04029	0.01291	-0.02387
		1.00000	4.63057	3.78981	0.46496	9.60828	-3.83439	13.30573	-4.42038	-12.83121	-4.11146	7.60191
			0.06454	0.05441	0.00224	0.13148	-0.05481	0.19003	-0.06320	-0.18332	-0.05865	0.08272
			1.00000	0.84304	0.03470	2.03718	-0.84924	2.94437	-0.97923	-2.84041	-0.90873	1.28168
				-0.74004	-0.00127	-0.00122	0.00061	-0.05612	0.00076	0.00220	0.00070	-0.79438
				1.00000	0.00171	0.00164	-0.00082	0.07583	-0.00102	-0.00297	-0.00094	1.07343
					0.01279	0.01640	0.00049	0.00017	-0.00053	-0.00097	-0.00030	0.02805
					1.00000	1.28225	0.03831	0.01329	-0.04143	-0.07584	-0.02345	2.19313
						0.00119	-0.00005	-0.00174	0.00133	0.00290	0.00075	0.00438
						1.00000	-0.04201	-1.46218	1.11764	2.43697	0.63025	3.68067
							-0.00065	0.00190	-0.00064	-0.00190	-0.00059	-0.00188
							1.00000	-2.92307	0.98461	2.92307	0.90769	2.89230
								0.00082	0.00228	0.00503	0.00134	0.00947
								1.00000	2.78084	6.13416	1.63414	11.54878
									-0.00800	-0.01764	-0.00469	-0.03033
									1.00000	2.20500	0.58625	3.79125
										0.00001	0.000008	0.000019
										1.00000	0.73404	1.73404
											-0.00001	-0.00001
											1.00000	1.00000

Table 17. The coded inverse matrix (Back solution)

C_1	C_2	C_3	C_4	C_5	C_6	C_7	C_8	C_9	C_{10}	C_{11}	C_{12}	Y_1(Oct.)
3310.8	340.9	13.4	-1355.7	-38.0	-922.37	360.1	12525.5	156.8	32810.8	-13623.5	-4667.0	97.300
	-79.9	-14.3	884.3	-84.0	5.82	203.7	1715.9	43.0	6343.2	-1969.2	-787.0	40.765
		-2.1	91.8	-10.0	3.88	-17.2	100.0	4.4	504.4	-99.4	-179.8	47.400
			-960.1	89.0	84.96	-65.2	-4065.9	-65.6	-13678.4	5522.0	-67.5	-205.120
				0.1	0.94	-0.3	44.3	-18.6	-98.7	36.7	-12.4	44.510
					13.92	14.1	-390.7	-11.5	-4666.2	1359.5	1233.1	-59.400
						23.3	-377.4	3.0	1325.8	17.2	-1139.4	-6.700
							45206.1	651.3	127622.8	-51606.8	-21647.4	0.415
								253.7	1152.4	-411.1	173.4	0.001
									324339.3	-119702.9	-103231.3	0.860
										34770.9	73404.2	-0.658
											-100000.0	-0.006

are given in table 17. Column 13 contains the coded values of the multiple regression coefficients (b_1', b_2', b_3' ...... b_{12}') for Y_1 (October). These coefficients are decoded by the formula:

$$b = b' \left(\frac{K_r}{K_c}\right) \quad . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \quad (12)$$

where, b = Decoded multiple regression coefficient.

b' = Coded multiple regression coefficient.

K_r = Coding factor for the row.

K_c = Coding factor for the column.

The decoded multiple linear regression coefficients (b_n) for all Y'x are listed in table 10. (Y_1, October is chosen for the computation example).

After the multiple linear regression coefficients b_1, b_2, b_3, ... b_{12} are determined, b_o is computed for any appropriate set of independent variables by solving the equation:

$$b_o N + b_1 \Sigma X_1 + b_2 \Sigma X_2 + \ldots\ldots + b_k \Sigma X_k = \Sigma Y \quad . \; . \; . \; . \; . \; . \; . \; . \; . \quad (13)$$

Table 11 gives the regression values of b_o.

Substitution of regression coefficients (for October) from table 10, table 11, and the independent values from table 12 (for 1957) in the forecasting equation yields the predicted runoff of October, 1957.

Y_1 = 8870.048 acre-feet (actual runoff was 8870 acre-feet)

RESULTS AND COMPARISON

The prediction equation (equation 8) and the computed coefficients (tables 10 and 11) have been used in making an actual streamflow prediction. The closeness of fit of the predicted runoff to that which occurred during the three years of record can be tested by comparing monthly the streamflow as computed by the equation with the actual streamflow (table 18). These results indicate that the use of soil moisture data in the prediction equation holds considerable promise. The equation must be tested by making some actual forecasts before final verification can be made.

Table 18. Error of computed monthly streamflow summarized for 36 months of record (1957-1959)

Percent of Error	Months	Percent of Total Months
0	18	50.00
1	3	8.33
2	3	8.33
6	1	2.78
7	1	2.78
9	1	2.78
11	1	2.78
12	1	2.78
14	1	2.78
15	1	2.78
17	4	11.10
19	1	2.78
Total	36	100.00

The U. S. Geological Survey classifies the general accuracy of its streamflow records in the following terms:

Excellent within 5 per cent

Good within 10 per cent

Fair within 15 per cent

Poor greater than 15 per cent

If these criteria may be applied to the results as indicated in table 18, the following conclusion may be drawn with regard to the accuracy of the fit:

24 months excellent

3 months good

4 months fair

5 months poor

The comparison of forecasts made by methods described herein with temperature-precipitation-snow survey method, Federal-State Cooperative published method and actual measured runoff are listed in table 19.

Since there is not enough data available, no attempt will be made to perform an analysis of variance of the individual variables considered in the forecasting equation.

Table 19. Comparison of computed streamflow made by methods described herein with temperature-precipitation-snow survey (11) method and published forecasts for the April-September streamflow.

Year	Months	Actual Flow (acre-ft.)	Soil Moisture Method: Runoff (acre-ft.)	Soil Moisture Method: Accuracy (percent)	Temp.-Pre.-Snow Method: Runoff (acre-ft.)	Temp.-Pre.-Snow Method: Accuracy (percent)	Published Forecasts: Runoff (acre-ft.)	Published Forecasts: Accuracy (percent)
1957	Oct.	8,870	8,870	100	3,408	26		
	Nov.	7,640	7,638	100	9,083	84	Published forecasts were not made on a monthly basis	
	Dec.	7,130	6,320	88	7,189	99		
	Jan.	6,530	6,668	98	6,222	95		
	Feb.	5,990	5,905	99	5,684	95		
	Mar.	7,230	7,230	100	6,281	87		
	Apr.	11,020	12,941	85	——	—		
	May	33,790	33,791	100	44,125	77		
	June	49,690	41,356	83	44,847	90		
	July	23,570	22,070	94	20,225	86		
	Aug.	13,720	13,658	100	12,905	94		
	Sept.	10,410	10,410	100	7,359	71		
Apr.-Sept. Flow		142,200	134,230	94	129,460	91	153,000	93
Annual Flow		185,600	176,840	95	167,330	90		

Table 19. Comparison of computed streamflow made by methods described herein with temperature-precipitation-snow survey (11) method and published forecasts for the April-September streamflow (continued).

Year	Months	Actual Flow (acre-ft.)	Soil Moisture Method Runoff (acre-ft.)	Soil Moisture Method Accuracy (percent)	Temp.-Pre.-Snow Method Runoff (acre-ft.)	Temp.-Pre.-Snow Method Accuracy (percent)	Published Forecasts Runoff (acre-ft.)	Published Forecasts Accuracy (percent)
1958	Oct.	9,740	9,740	100	7,881	81		
	Nov.	8,260	8,263	100	9,121	91	Published forecasts were not made on a monthly basis	
	Dec.	7,630	6,802	89	7,496	98		
	Jan.	6,730	6,668	98	6,977	96		
	Feb.	6,210	6,302	99	6,171	99		
	Mar.	6,990	6,990	100	6,859	98		
	Apr.	12,800	11,961	93	20,401	63		
	May	47,200	47,198	100	38,441	81		
	June	38,940	41,356	83	39,688	98		
	July	17,630	16,131	91	18,911	93		
	Aug.	12,370	12,450	99	12,517	99		
	Sept.	9,810	9,810	100	9,924	99		
Apr.-Sept. Flow		138,770	138,910	100	139,880	99	163,000	85
Annual Flow		184,300	183,670	100	184,390	100		

Table 19. Comparison of computed streamflow made by methods described herein with temperature-precipitation-snow survey (11) method and published forecasts for the April-September streamflow (continued).

Year	Months	Actual Flow (acre-ft.)	Soil Moisture Method Runoff (acre-ft.)	Soil Moisture Method Accuracy (percent)	Temp.-Pre.-Snow Method Runoff (acre-ft.)	Temp.-Pre.-Snow Method Accuracy (percent)	Published Forecasts Runoff (acre-ft.)	Published Forecasts Accuracy (percent)
1959	Oct.	8,580	8,580	100	11,497	75		
	Nov.	7,690	7,688	100	8,897	86	Published forecasts were not made on a monthly basis	
	Dec.	7,060	8,716	81	7,360	96		
	Jan.	6,270	6,407	98	6,894	91		
	Feb.	5,560	5,553	100	6,173	90		
	Mar.	6,510	6,510	100	7,853	83		
	Apr.	12,600	10,519	83	4,363	35		
	May	25,700	25,710	100	31,117	83		
	June	29,940	34,626	86	25,182	84		
	July	14,910	17,909	83	15,980	93		
	Aug.	10,420	10,400	100	10,580	98		
	Sept.	8,400	8,400	100	7,973	95		
Apr.-Sept. Flow		101,970	107,560	95	95,200	93	116,000	88
Annual Flow		143,640	151,000	95	143,870	100		

THE 1961 STREAMFLOW FORECAST

The 1961 streamflow forecast is presented to illustrate the complete forecasting procedure and to allow future evaluation of the forecasting methods.

The independent variables were obtained from Fourier coefficients, equations (6) and (7); see also tables 20 and 21. The prediction data are not available for the April-September period, in this case the long-time monthly means are used. The regression coefficients for these computations are the same as shown in tables 10 and 11.

The solution of equation (8) for Y is the final step in the forecasting procedure. For substitution in this equation, the appropriate regression coefficients are chosen from tables 10 and 11, and the independent variables from table 22. The forecasted monthly streamflow are given as follows:

Months	Forecast flow (acre feet)	Actual flow (acre feet)	Percent of Accuracy
October	7,927	7,140	90.07
November	7,162	6,580	91.87
December	9,334	5,930	63.53
January	5,762	5,580	96.84
February	13,670	4,880	35.70
March	4,935	5,430	90.88
April	57,297		
May	18,105		
June	35,417		
July	12,356		
August	97,284		
September	3,986		

The results of the forecast indicate that the predicted flow for February, April, and August are excessively high. However, the predicted flow of other months have high degree of accuracy as indicated by comparison with actual flow. Because of the short record and limited time, only three years soil moisture data were available for this study. The variation in soil moisture, snow cover, and runoff for the three year period Fourier coefficient and regression coefficient were small. Consequently, revisions in the coefficients will be necessary as a longer record of soil moisture becomes available. The 1961 forecast was included to illustrate how the soil moisture data can be utilized in making streamflow forecasts.

Table 20. Computation of Fourier streamflow coefficients (Streamflow on Logan River, Utah)

Month	X	(A_{F_1})	Coeff. cos θ	(A_{F_2})	Coeff. sin θ	(A_{F_3})	Coeff. cos 2θ	(A_{F_4})	Coeff. sin 2θ
		cosθ	xcosθ	sinθ	xsinθ	cos2θ	xcos2θ	sin2θ	xsin2θ
Oct.	7,140	1.000	7,140	0.000	0	1.000	7,140	0.000	0
Nov.	6,580	0.866	5,698	0.500	3,290	0.500	3.290	0.866	5,698
Dec.	5,930	0.500	2,965	0.866	5,135	-0.500	-2,965	0.866	5,135
Jan.	5,580	0.000	0	1.000	5,580	-1.000	-5,580	0.000	0
Feb.	4,880	-0.500	-2,440	0.866	4,226	-0.500	-2,440	-0.866	-4,226
Mar.	5,430	-0.866	-4,702	0.500	2,715	0.500	2,715	-0.866	-4,702
Apr.	15,400	-0.100	-15,400	0.000	0	1.000	15,400	0.000	0
May	35,710	-0.866	-30,925	-0.500	-17,855	0.500	17,855	0.866	30,925
June	36,460	-0.500	-18,230	-0.866	-31,574	-0.500	-18,230	0.866	31,574
July	19,230	0.000	0	-1.000	-19,230	-1.000	-19,230	0.000	0
Aug.	12,600	0.500	6,300	-0.866	-10,912	-0.500	-6,300	-0.866	-10,912
Sept.	9,850	0.866	8,530	-0.500	-4,925	0.500	4,925	-0.866	-8,530
Sum	164,790		-41,064		-63,550		-3,420		44,962
A_F	13,731		-6,844		-10,592		-570		7,494

Table 21. Computation of Fourier soil moisture coefficients
(Soil moisture at Klondike Narrows, Utah)

Month	X	(A_{M_1})	Coeff. cos θ	(A_{M_2})	Coeff. sin θ	(A_{M_3})	Coeff. cos 2θ	(A_{M_4})	Coeff. sin 2θ
		cosθ	xcosθ	sinθ	xsinθ	cos2θ	xcosθ	sin2θ	xsin2θ
Oct.	5.36	1.000	5.37	0.000	0	1.000	5.37	0.000	0
Nov.	5.69	0.866	4.93	0.500	2.85	0.500	2.85	0.866	4.93
Dec.	7.31	0.500	3.66	0.866	6.33	-0.500	-3.66	0.866	6.33
Jan.	7.87	0.000	0	1.000	7.87	-1.000	-7.87	0.000	0
Feb.	8.24	-0.500	-4.12	0.866	7.14	-0.500	-4.12	-0.866	-7.14
Mar.	8.60	-0.866	-7.45	0.500	4.30	0.500	4.30	-0.866	-7.45
Apr.	9.58	-1.000	-9.58	0.000	0	1.000	9.58	0.000	0
May	10.05	-0.866	-8.70	-0.500	-5.03	0.500	5.03	0.866	8.70
June	10.92	-0.500	-5.46	-0.866	-9.46	-0.500	-5.46	0.866	9.46
July	8.99	0.000	0	-1.000	-8.99	-1.000	-8.99	0.000	0
Aug.	5.63	0.500	2.82	-0.866	-4.88	-0.500	-2.82	-0.866	-4.88
Sept.	4.66	0.866	4.04	-0.500	-2.33	0.500	2.33	-0.866	-4.04
Sum	92.91		-14.49		-2.20		-3.46		5.91
A_M	7.743		-2.415		-0.367		-0.577		0.985

Table 22. The soil moisture Fourier coefficients, streamflow Fourier coefficients, and snow water content used in the 1961 streamflow forecast.

April 1 water content of snow at 2 snow courses (in.)
(Franklin Basin X_1, and Garden City Summit X_2)

X_1	X_2
21.0	12.9

Klondike Narrows annual monthly mean soil moisture and Fourier coefficients.

$\bar{M}$ (in.)	A_{M_1}	A_{M_2}	A_{M_3}	A_{M_4}
7.743	-2.415	-0.367	-0.577	0.985

Logan River annual monthly mean streamflow and Fourier coefficients.

$\bar{F}$ (acre ft.)	A_{F_1}	A_{F_2}	A_{F_3}	A_{F_4}
13,731	-6,855	-10,592	-570	7,494

CONCLUSIONS

1. The closeness of fit of the prediction equation with the actual streamflow indicate that the use of soil moisture data in the prediction equation may hold considerable promise.

2. The utilization of the soil moisture data may help to correct discrepancies between forecasts made from snow survey data and precipitation, particularly in those years where there is considerable winter snow melt.

3. Since only three years soil moisture data, with not much variation, are available for this study, the results of the 1961 forecast indicate data are insufficient yet to be utilized in the forecasting equation. Revisions in the coefficients will be necessary as a longer record of soil moisture becomes available.

4. Streamflow forecasting by the statistical approach used in this thesis is quick and simple after the relationships have been determined by multiple regression. The initial computation for determining the coefficients with a desk electric calculator are laborious, but can be accomplished rather quickly with highspeed computation procedures and machines.

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APPENDIX

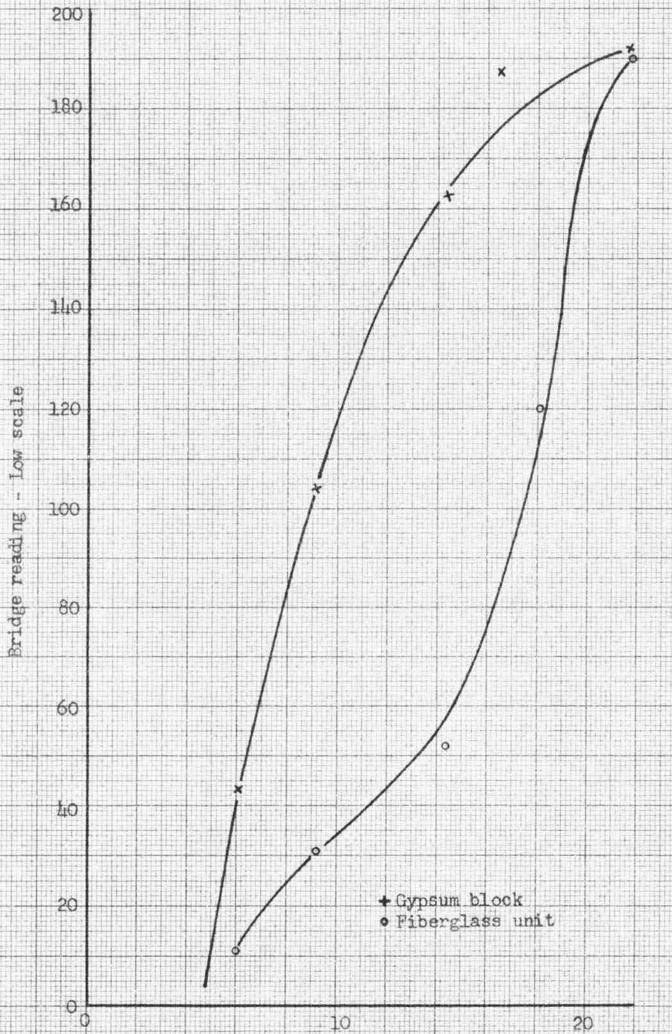

Figure 1. Calibration curve for Klondike Narrows Station hole 1, 1st foot soil water content versus bridge reading.

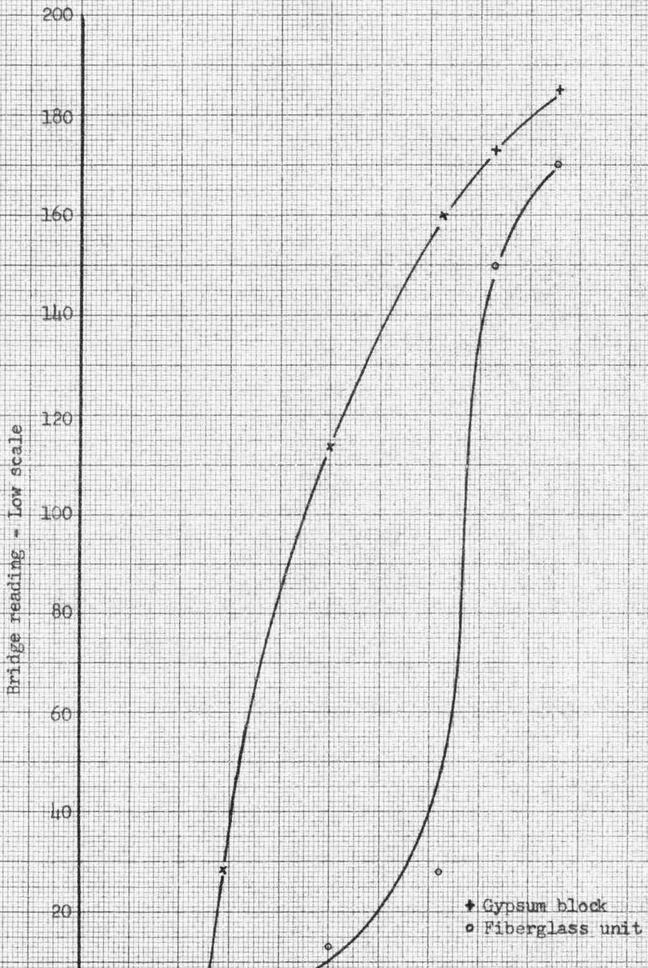

Figure 2. Calibration curve for Klondike Narrows Station hole 1, 2nd foot soil water content versus bridge reading.

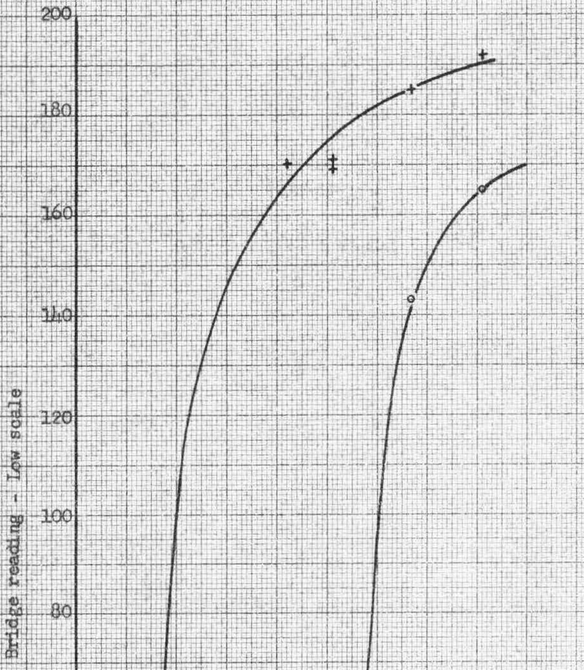

Figure 3. Calibration curve for Klondike Narrows Station hole 1, 3rd foot soil water content versus bridge reading.

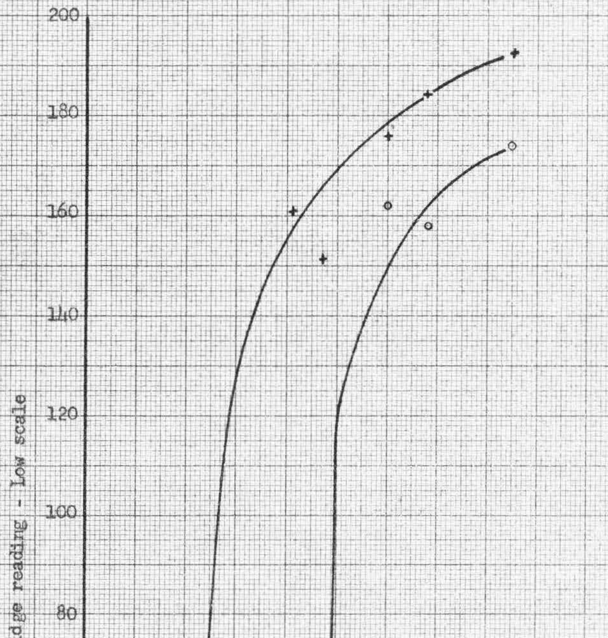

Figure 4. Calibration curve for Klondike Narrows Station hole 1, 4th foot soil water content versus bridge reading.

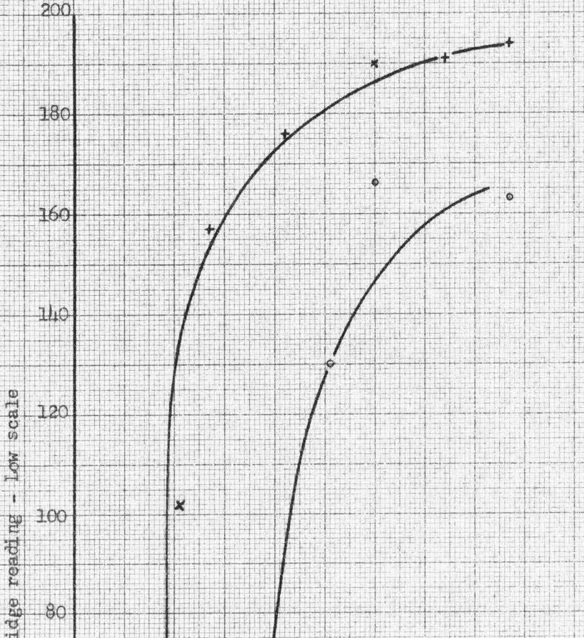

Figure 5. Calibration curve for Klondike Narrows Station hole 1, 5th foot soil water content versus bridge reading.

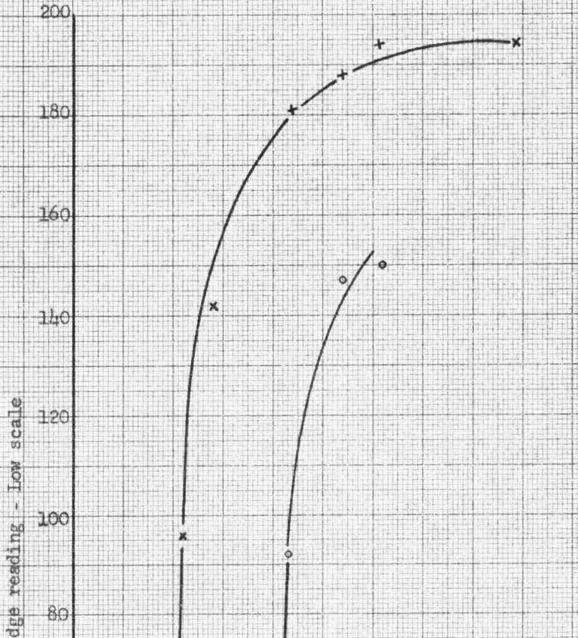

Figure 6. Calibration curve for Klondike Narrows Station hole 1, 6th foot soil water content versus bridge reading.

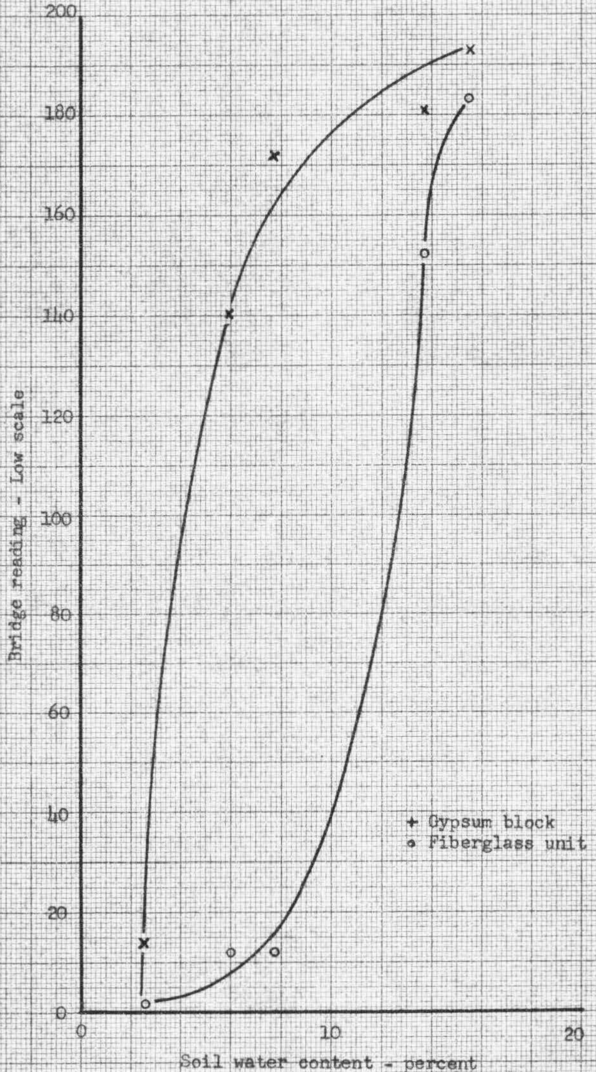

Figure 7. Calibration curve for Klondike Narrows Station hole 2, 1st foot soil water content versus bridge reading.

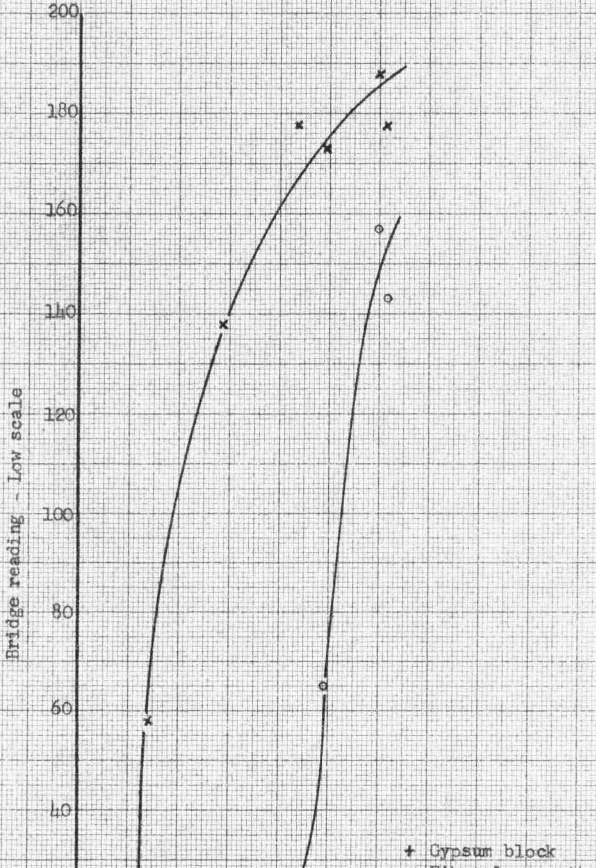

Figure 8. Calibration curve for Klondike Narrows Station hole 2, 2nd foot soil water content versus bridge reading.

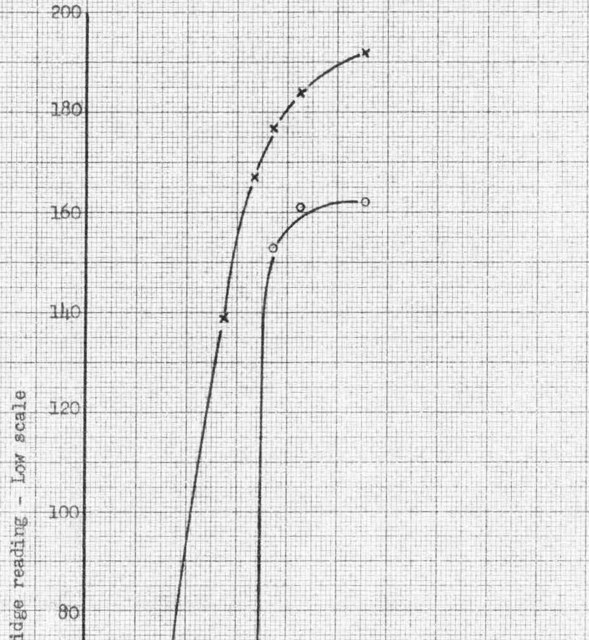

Figure 9. Calibration curve for Klondike Narrows Station hole 2, 3rd foot soil water content versus bridge reading.

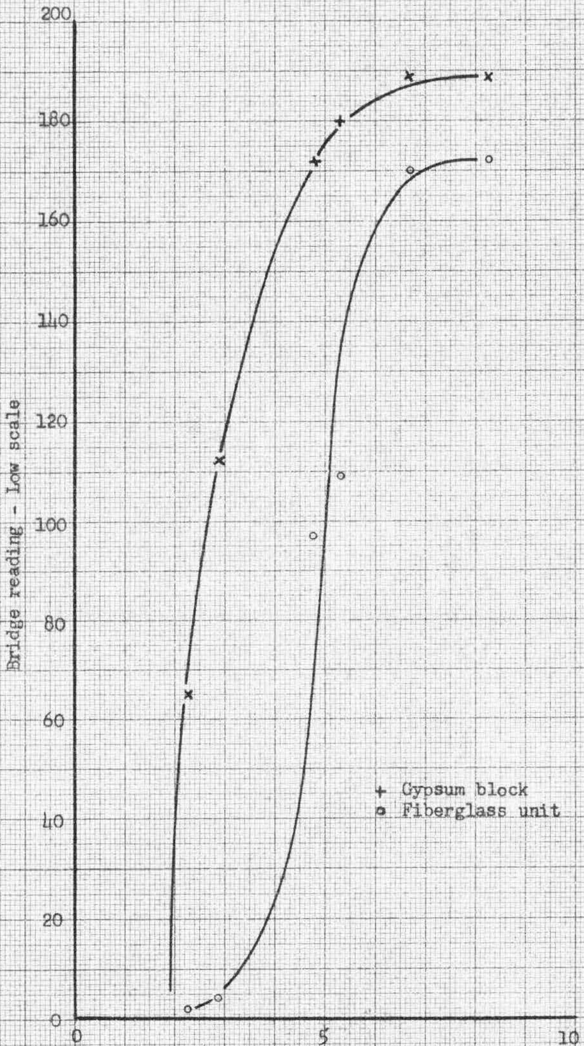

Figure 10. Calibration curve for Klondike Narrows Station hole 2, 4th foot soil water content versus bridge reading.

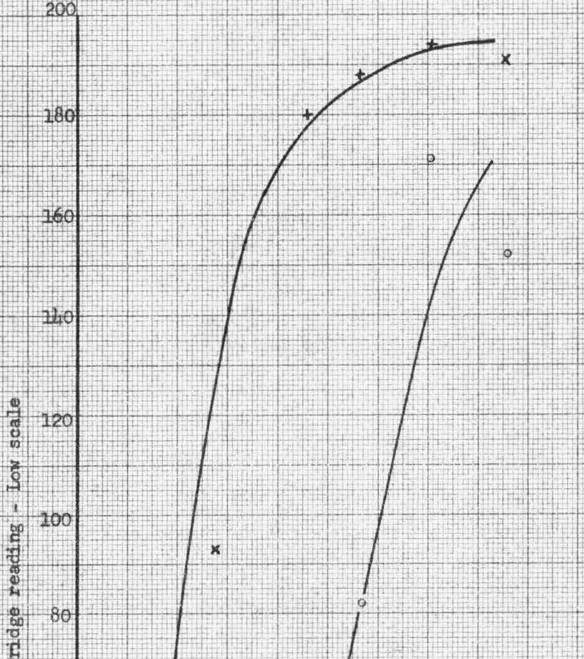

Figure 11. Calibration curve for Klondike Narrows Station hole 2, 5th foot soil water content versus bridge reading.

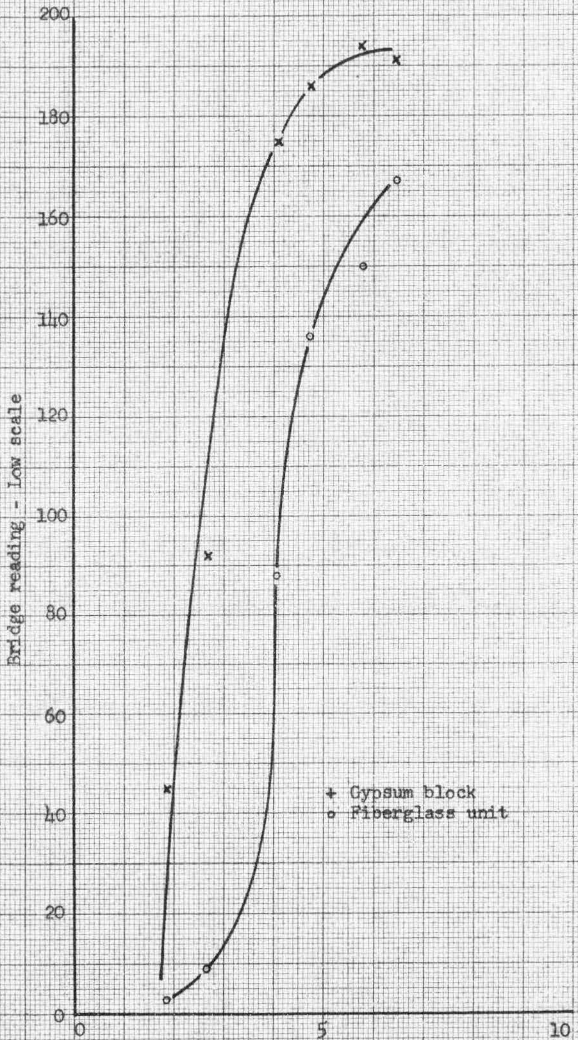

Figure 12. Calibration curve for Klondike Narrows Station hole 2, 6th foot soil water content versus bridge reading.

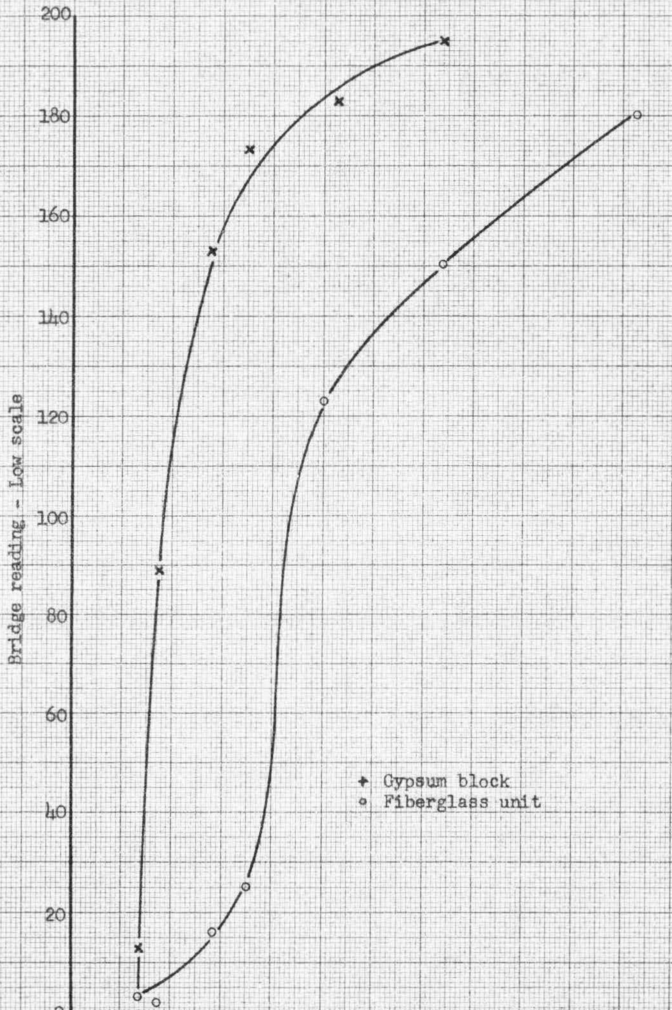

Figure 13. Calibration curve for Klondike Narrows Station hole 3, 1st foot soil water content versus bridge reading.

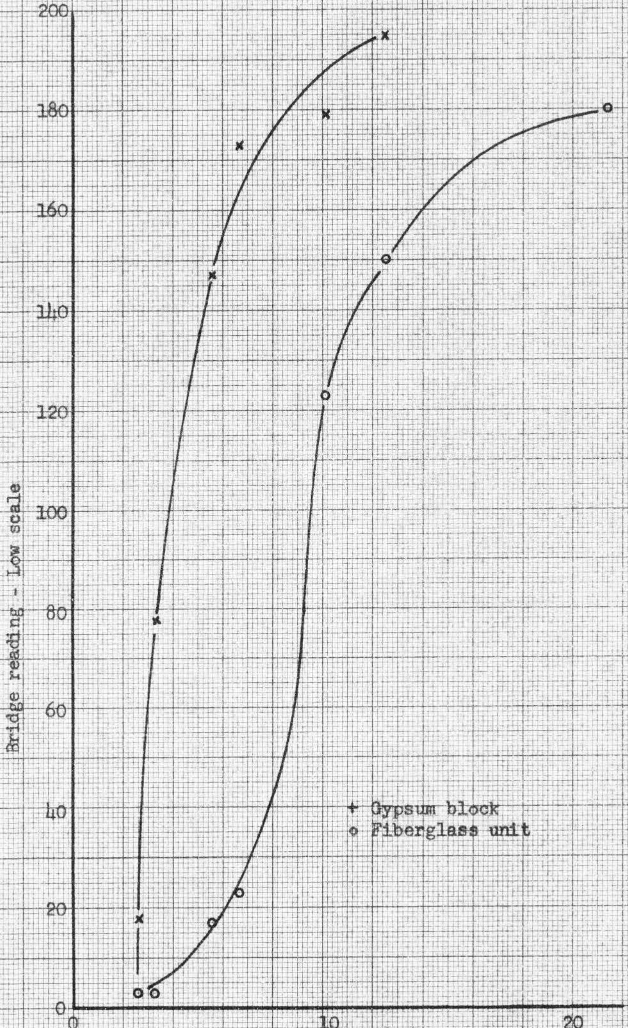

Figure 14. Calibration curve for Klondike Narrows Station hole 3, 2nd foot soil water content versus bridge reading.

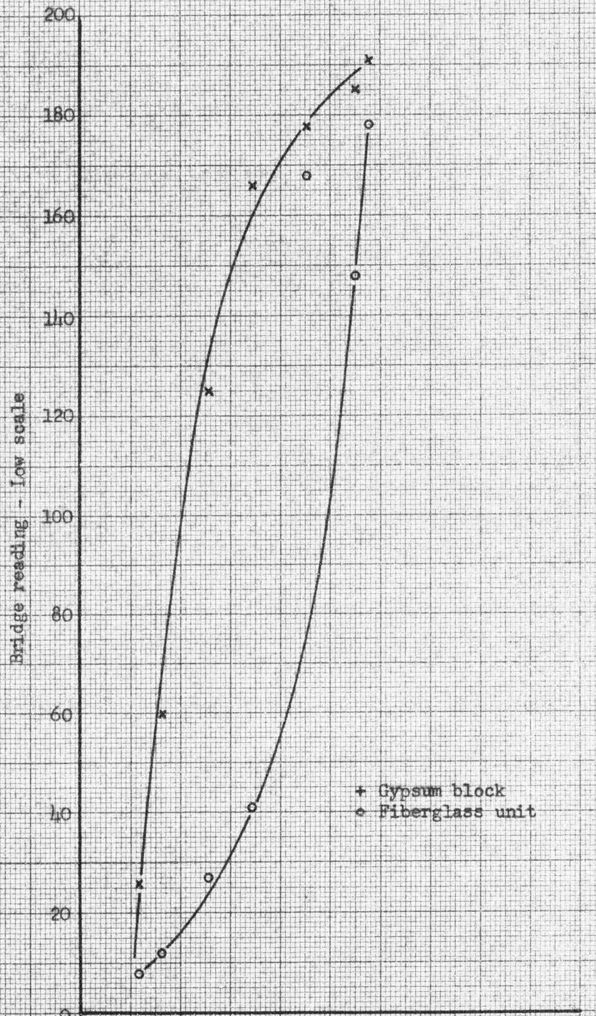

Figure 15. Calibration curve for Klondike Narrows Station hole 3, 3rd foot soil water content versus bridge reading.

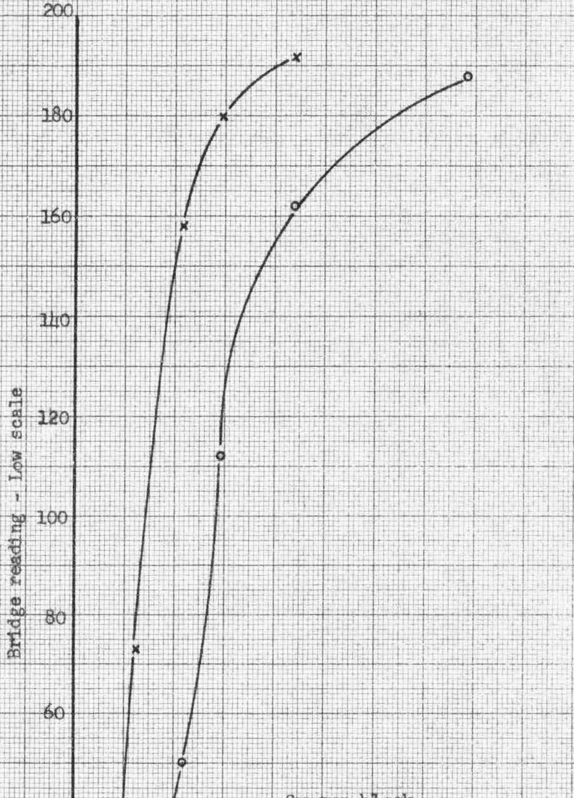

Figure 16. Calibration curve for Klondike Narrows Station hole 3, 4th foot soil water content versus bridge reading.

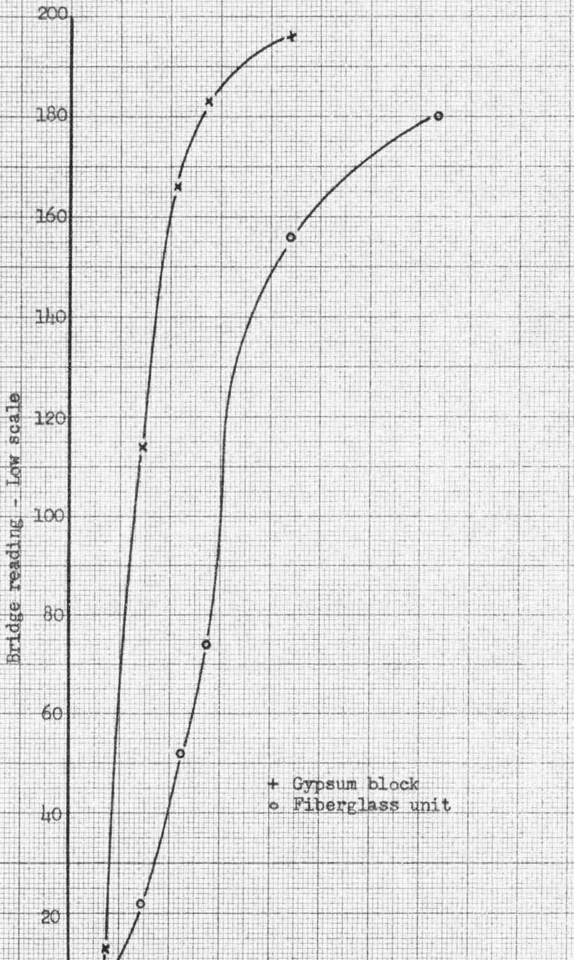

Figure 17. Calibration curve for Klondike Narrows Station hole 3, 5th foot soil water content versus bridge reading.

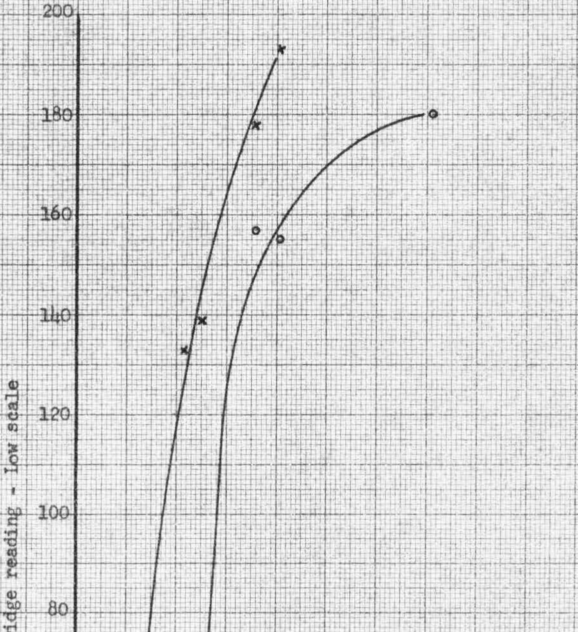

Figure 18. Calibration curve for Klondike Narrows Station hole 3, 6th foot soil water content versus bridge reading.